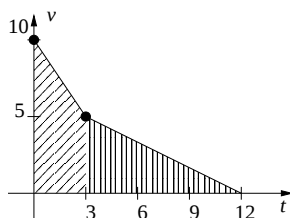


1. A rapidly traveling red-coated bicycle-borne mathematics professor is observed to decelerate from 10 m/s to 5 m/s in 3 seconds. Suppose that the mathematician travels 45 m before stopping (without speeding up). Carrie the Calculus Student thinks the professor stopped in 6 seconds, while Colin thinks it took 12 seconds. Is either of these two careful considerations of calculus computation clearly correct (or clearly incorrect)? Could the professor have taken less than 6 seconds or more than 12 seconds to stop? (4 points)

Solution: We know the distance-velocity data

t	0	3	6
v	10	5	?

So if we use a left-hand sum to estimate the distance traveled in the first six seconds, we get distance = $\int_0^6 v(t) dt = (3)(10) + (3)(5) = 45$ m. So Carrie's estimate is a reasonable one, and, because we know that the velocity is never increasing, it must be an overestimate for the distance traveled in the first six seconds. So the minimum time the professor could have taken to stop is six seconds. However, the velocity might not go to zero at $t = 6$. For example, if the velocity graph looks like the following,



then we can calculate the exact distance traveled by finding areas under the curve: distance = $\int_0^3 v(t) dt + \int_3^{12} v(t) dt = ((3)(5) + (1/2)(3)(5)) + (1/2)(9)(5) = 45$ m. So Colin's estimate is also reasonable. Of course, the velocity could also get much smaller faster after $t = 5$, in which case the time to stop would be larger than 12 s, so this is not the maximum possible time.

2. Suppose that the rate at which assignments appear as a function of the number of weeks, t , a student finds her or himself into the semester is given by $r(t) = e^{0.19t}$. How many assignments will this student get in the last week of class of the semester (the semester has 14 weeks of classes)? (3 points)

Solution: We know that the total change in the number of objects is equal to the integral of the rate, so here total assignments = $\int_{13}^{14} r(t) dt = \int_{13}^{14} e^{0.19t} dt$. We can evaluate this with a calculator, finding the total number of assignments to be 13.02.

3. A function $f(x)$ is shown in the figure to the right. If $g'(x) = f(x)$, what can you say about $g(0)$? $g(1)$? $g(3) - g(1)$? (3 points)

Solution: Because $g'(x) = f(x)$, we know that $g(b) - g(a) = \int_a^b f(x) dx$, by the Fundamental Theorem of Calculus. And we know that the integral of $f(x) dx$ is just the area between the graph $y = f(x)$ and the x -axis. Thus we can't say anything about $g(0)$, but we know that $g(1)$ is $1/2$ a unit larger than the value of $g(0)$. And similarly, $g(3) - g(1) = \int_1^3 f(x) dx = (1/2)(1)(2) + (1/2)(2/3)(2) - (1/2)(1/3)(1) = \frac{3}{2}$.

