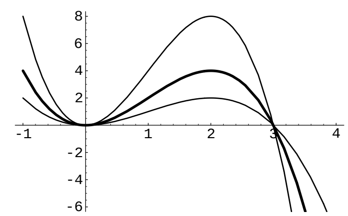
MATH 115-018 QUIZ 4 / 27 Jan 2005 Name:______________________________

1. (2 points) Find an equation of a polynomial of minimal degree that could produce the graph to the right. Could you find any other equation that would produce the graph? Explain.



Solution: The polynomial turns twice, once at $(0, 0)$ and once between $x = 0$ and $x = 3$, so it must be at least a polynomial of degree three (that is, a cubic). We also note that it has a double zero at $x = 0$, so it must have a factor of $(x - 0)^2 = x^2$, and it has another zero at $x = 3$, so it must have a factor of $(x - 3)$. This gives us a cubic, so we're close to done. The one other thing to note is that as $x \to -\infty$, $f(x) \to \infty$, and as $x \to \infty$, $f(x) \to -\infty$—which is the opposite of what a normal cubic would do (because a normal cubic looks like $f(x) \sim x^3$). So we must have $f(x) = -x^2(x - 3)$. This isn't unique, however—any constant (positive) multiple of this will stretch or scrunch it vertically, which doesn't change the zeros. The figure to the right, above, shows this—we've drawn in $y = -\frac{1}{2}\, x^2(x - 3)$ and $y = -2x^2(x - 3)$ in addition to $y = -x^2(x - 3)$.

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2. (3 points) Consider the function $f(x)$ given by

$$f(x) = \begin{cases} x + 8, & x < a \\ 2x^2 + x, & x \geq a. \end{cases}$$

Find, if possible, a value for a that will make $f(x)$ continuous on any interval on the real number line.

Solution: Clearly $f(x)$ is continuous for any interval that lies to the left of $x = a$, and similarly is continuous for any interval to the right of $x = a$. Thus the only point we are concerned with is $x = a$. If x approaches a from the left (that is, through values that are less than a), the left-hand side of the equation, $f(x) = x + 8$, approaches the value $a + 8$. Similarly, if x approaches a from the right, $f(x)$ approaches the value $2a^2 + a$. If $f(x)$ is to be continuous, these two values must be the same, so $2a^2 + a = a + 8$, or $2a^2 = 8$, so $a^2 = 4$ and $a = \pm 2$. Thus setting a to either of the values $a = -2$ or $a = 2$ will result in $f(x)$ being continuous on any interval.

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3. (3 points) Sketch a possible graph of your position, $s(t)$ (relative to the fine dining establishment indicated below), as a function of time, t, if your position is described by the following scenario: "After finishing lunch at a fine dining establishment, you proceed at a brisk but steady pace in a straight line to your favorite class, calculus. Halfway there, you realize that you might be late, and pick up your pace so that by the time you get to class you are sprinting along at a great velocity." Mark on your graph points or sections of the graph which illustrate the different parts of the scenario indicated.

Solution: A possible graph is shown to the right. We start at the point $(0, \text{dining establishment})$ (the black point on the graph) and proceed with a steady velocity, so the slope of the position graph is constant (it is a straight line). Then at the open point, $(\text{might be late}, \text{halfway there})$, we pick up the pace, so the slope of the graph increases. The slope (our velocity) continues increasing until we get to the point $(\text{classtime}, \text{at class})$, where the slope is really steep because we're going with "great velocity." Note that "halfway there" refers to the *distance*, that is, the vertical axis of the graph, not the time. Also note that the slope of the graph shouldn't ever decrease—the scenario says that we proceed steadily and then speed up.