MATH 115-018 QUIZ 5 / 3 Feb 2005 Name:______________________________

1. (2 points) What is a good estimate for $\lim_{x\to 0} \frac{2^x-1}{x}$? Give your answer to at least three decimal places, and explain in a sentence how you know it is that accurate.

Solution: The limit means that we let x get smaller and smaller, eventually being so small as to be infinitessimal. Plugging in smaller and smaller values of x, we get the following table of values.

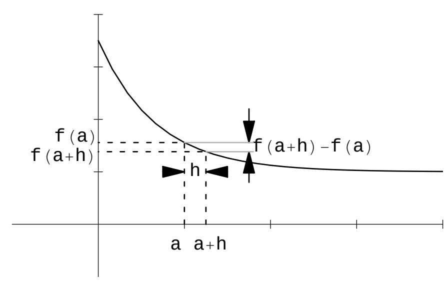
$x =$	0.1	0.01	0.001	0.0001
$\frac{2^x-1}{x} =$	0.717735	0.695555	0.693387	0.693171
$x =$	-0.1	-0.01	-0.001	-0.0001
$\frac{2^x-1}{x}$	0.669670	0.690750	0.692907	0.693123

We see that for all $|x| \leq 0.001$ the corresponding values of $\frac{2^x-1}{x}$ are, to three decimal places, equal to 0.693. We therefore conclude that $\lim_{x\to 0} \frac{2^x-1}{x} = 0.693$.

2. (3 points) What is the limit definition for the derivative of a function $f(x)$ at the point $x = a$? Illustrate the parts of the definition on the figure shown below, to the right.

Solution: The limit definition fo the derivative of $f(x)$ at $x = a$ is

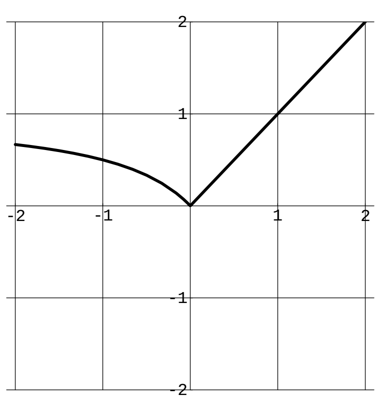
$$f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

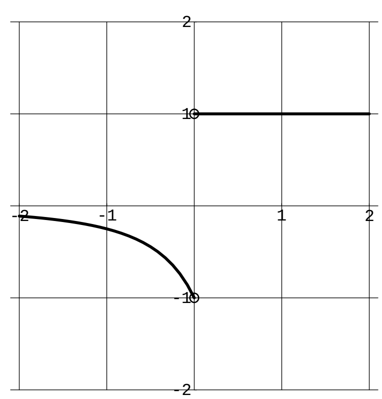


We can show the parts of this on the graph to the right as follows: $x = a$ and $x = a+h$ are two points on the x-axis, and $f(a)$ and $f(a+h)$ are the corresponding y-coordinates on the graph of $f(x)$. Then $f(a+h) - f(a)$ is the vertical distance between the y-coordinates, and h the horizontal distance between the x-coordinates. Thus $\frac{f(a+h)-f(a)}{h}$ is just the slope of the line between those two points, and as we let h go to zero the points $x = a+h$ and $x = a$ come together so that we end up with the slope of the curve at the point $x = a$, which is $f'(a)$.

3. (3 points) If the graph to the left below shows $f(x)$, sketch in the axes to the right $f'(x)$.

Solution:





Note that the derivative is *undefined* at $x = 0$, because of the sharp corner there.