MATH 115-018 QUIZ 6 / 17 Feb 2005 Name:____________________________

NOTE: This quiz was a bit long and none-too-easy, so I marked it out of six points rather than 8.

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1. (4 points) Find derivatives of each of the following functions.

(a) $f(x) = (3x+2)\cdot 3^x$ **(b)** $z(y) = \sqrt{y^5 - e} - \sin(2)$ **(c)** $g(\theta) = \frac{4}{e^{7\theta}}$ **(d)** $r(t) = \pi\, 4^{(t+1)/t}$

Solution:

(a) $f'(x) = 3\cdot 3^x + (3x+2)\ln(3)\cdot 3^x$.
(b) $z'(y) = \frac{1}{2}(y^5 - e)^{-1/2}\cdot 5y^4$.
(c) Note that $g(\theta) = 4e^{-7\theta}$, so that $g'(\theta) = -28e^{-7\theta}$. Alternately, using the quotient rule,

$$g'(\theta) = \frac{0\cdot e^{7\theta} - 4\cdot e^{7\theta}\cdot 7}{(e^{7\theta})^2}.$$

(d) Note that $r(t) = \pi\, 4^{1+t^{-1}}$, so that $r'(t) = \pi\ln(4)\cdot 4^{1+t^{-1}}\cdot(-t^{-2})$. Alternately, without simplifying the exponent, and using the quotient rule,

$$r'(t) = \pi\ln(4)\cdot 4^{(t+1)/t}\cdot\left(\frac{1\cdot t - (1+t)\cdot 1}{t^2}\right).$$

2. (2 points) The following table gives values for $f(x)$, $f'(x)$, $g(x)$, and $g'(x)$ at different values of x. If $h(x) = g(f(x))$, find $h'(2)$. Be sure that it is clear how you obtain your answer (simply writing something like "8" will get no credit).

$x =$	-2	-1	0	1	2
$f(x) =$	2	0	1	-2	-1
$f'(x) =$	-1	1	2	0	-2
$g(x) =$	1	2	-2	-1	0
$g'(x) =$	0	-2	-1	2	1

Solution: Using the chain rule, $h'(2) = g'(f(2))\cdot f'(2) = g'(-1)\cdot(-2) = (-2)(-2) = 4$. It's possible to get an estimate by finding values for $h(1)$ and $h(2)$ and using a difference quotient, but this isn't as accurate. (Let's find this: $h(1) = g(f(1)) = g(-2) = 1$ and $h(2) = g(f(2)) = g(-1) = 2$, so $h'(2) \approx \frac{h(2)-h(1)}{2-1} = \frac{2-1}{2-1} = 1$.)

3. (2 points) For what values of x is the *derivative* of the function $p(x) = x\cdot 3^x$ *decreasing*?

Solution: The derivative $p'(x)$ is decreasing when the second derivative $p''(x)$ is negative. Calculating these derivatives, we have $p'(x) = 3^x + x\cdot\ln(3)\cdot 3^x$, so that $p''(x) = 2\ln(3)\cdot 3^x + x\cdot\ln(3)^2\cdot 3^x = \ln(3)\cdot 3^x\cdot(2 + x\ln(3))$. Note that $\ln(3)\approx 1.099 > 0$ and $3^x > 0$ for all x. Thus $p''(x) < 0$ if $2 + x\ln(3) < 0$, which is when $x\ln(3) < -2$, or $x < -\frac{2}{\ln(3)}$ (which is approximately $x < -1.82$, which we could also find by graphing $p''(x)$).

Another solution is to find $p'(x) = 3^x + x\cdot\ln(3)\cdot 3^x$ and then look for where this function is decreasing, but it's hard to get an accurate sense of where this is.