MATH 115-018 QUIZ 9 / 17 Mar 2005 Name:______________________________

1. (4 points) Let $f(x)$ be a function that is everywhere differentiable. Suppose that you know the values for $f'(x)$ given in the table below.

$x =$	-3	-2	-1	0	1
$f'(x) =$	2	0.5	-0.5	-1	-0.5

a. Identify the location of any critical points and local maxima or local minima, if any, that this data indicates $f(x)$ has.

b. If possible, identify the location of any inflection points of $f(x)$, and the concavity of the graph of $f(x)$. If it is not possible, briefly explain why.

Solution: **a.** We know that $f(x)$ has critical points at points where $f'(x)$ is zero or undefined. Because $f(x)$ is everywhere differentiable there are no undefined points, and given that $f'(-2) = 0.5$ and $f'(-1) = -0.5$ we know that there is a point between $x = -2$ and $x = -1$ where $f'(x) = 0$. So there is a critical point between these two x-values. Because the sign of $f'(x)$ changes from positive to negative there, we know that this is a local maximum.

b. Inflection points occur where $f(x)$ changes concavity, which is where $f'(x)$ has a local maximum or minimum. From the data it is clear that $f'(x)$ has a local minimum between $x = -1$ and $x = 1$, so there is an inflection point in the graph of $f(x)$ between those two x-values. Because $f''(x)$ is negative where $f'(x)$ is decreasing, we know that $f(x)$ is concave down before the inflection point and concave up thereafter.

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2. (4 points) Consider the family of functions given by $y = a\ln(x) + bx^2$, with a and b both positive. If the graph of a member of this family has an inflection point at $x = 3$, what can you say about a and b?

Solution: We know that an inflection point is where $\frac{d^2y}{dx^2}$ $(= f''(x))$ changes sign. Here, $\frac{dy}{dx} = \frac{a}{x} + 2bx$ and $\frac{d^2y}{dx^2} = -\frac{a}{x^2} + 2b$. Inflection points will occur when $\frac{d^2y}{dx^2} = 0$, which is $-\frac{a}{x^2} + 2b = 0$, or $x = \sqrt{\frac{a}{2b}}$ †. ("Clearly"‡ as x goes through $\sqrt{\frac{a}{2b}}$, $\frac{a}{x^2}$ changes from greater than to less than $2b$, or vice versa, so $\frac{d^2y}{dx^2}$ must change sign at both of these points. Thus they must actually be inflection points.)

Then, if the family is to have an inflection point at $x = 3$, we must have $3 = \sqrt{\frac{a}{2b}}$, or, $9 = \frac{a}{2b}$. Thus the condition $a = 18b$ will result in members of the family having an inflection point at $x = 3$.

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† *Really it's* $x = \pm\sqrt{\frac{a}{2b}}$, *but because we started with a term involving* $\ln(x)$ *the negative value doesn't make sense.*

‡ *Check to make sure that this makes sense to you—try plugging in some values for a and b if you're not sure about it.*