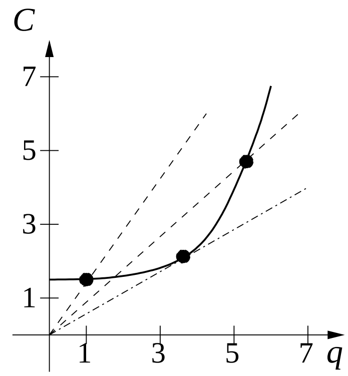
MATH 115-018 QUIZ 10 / 24 March 2005 Name:________________

1. (3 points) Suppose that the cost C (in 100s of dollars) of manufacturing q widgets is given as a function of q (in 100s of widgets) by the graph to the right. If the widget manufacturing company wants to minimize the *cost per widget*, (approximately) how many widgets should it make? Why?



Solution: The cost per widget is $R = \frac{C}{q}$. Because this is the same as $R = \frac{C-0}{q-0}$, R is represented on the graph as the slope of a line through the origin and through a point on the graph of $C(q)$. Three such lines are shown in the figure. Clearly the minimum slope of such a line will occur when the line is tangent to the curve $C(q)$, as shown by the dash-dotted line. This occurs for $q \approx 3.75$ (that is, 375 widgets).

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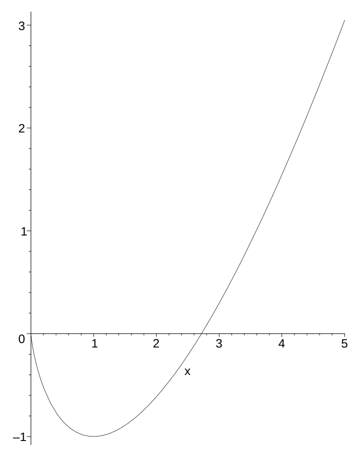
2. (3 points) Use calculus to find the absolute maximum and minimum values of the function $f(x) = x\ln(x) - x$ on $0 < x \leq 5$.

Solution: We know the absolute maximum and minimum occur at critical points or at endpoints. Here $f'(x) = \ln(x) + 1 - 1 = \ln(x)$, so the only critical point is $x = 1$. Comparing this value with the values at the ends of the interval, we see $f(1) = -1$, $f(5) = 5\ln(5) - 5 \approx 3.047$, and by examining the graph of $f(x)$ (shown to the right), as $x \to 0$, $f(x) \to 0$. Thus the absolute minimum value of $f(x)$ is -1, at $x = 1$, and the absolute maximum is $5\ln(5) - 5$, at $x = 5$.



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3. (2 points) A hot-air balloon is launched from a patch of flat ground on a windy day. As it rises, the wind pushes it at a 45° angle towards a group of onlookers who are standing 50 feet away. Imagining that you might want to find the minimum distance from the onlookers to the balloon, write an equation that gives that distance. Be sure that it is clear what your variables represent. *Do Not Actually Find The Minimum Distance!*

Solution: The situation we're considering is shown to the right. The balloon's path is given by $y = x$, so that for any horizontal distance x that it has traveled, it will be at at the point (x, x). The onlookers are at the point $(50, 0)$. Therefore the legs of the right triangle shown have lengths $50 - x$ and x, and the distance is given by $d(x) = \sqrt{(50 - x)^2 + x^2}$.