MATH 115-018 QUIZ 12 / 14 April 2005 Name:_____

NOTE: You have 12 MINUTES to work on this quiz. At that point, your work will be collected, and you will have 10 MINUTES to work with your team on it. Your final grade will be a combination of the scores on the work on the two efforts.

1. (3 points) Suppose that $\int_1^3 g(x)\,dx = 5$, $f(x)$ is given by the table below, that $F'(x) = f(x)$, and that $F(2) = 4$. Find $\int_1^3 (2f(x) - g(x))\,dx$.

$x =$	1	1.5	2	2.5	3
$f(x) =$	2	3	5	4	4

Solution: We know that $\int_1^3 (2f(x) - g(x))\,dx = 2\int_1^3 f(x)\,dx - \int_1^3 g(x)\,dx$. Using a left hand sum, we can estimate $\int_1^3 f(x)\,dx \approx \frac{1}{2}(2+3+5+4) = 7$. Thus $\int_1^3 (2f(x) - g(x))\,dx \approx 2 \cdot 7 - 5 = 9$.

2. (3 points) If the sleeplessness rate of students is given by $r(t) = t\sin(t)$, in missed hours/night (with t =weeks into the semester), estimate how many hours of sleep a student misses in the course of the semester (which is about 14 weeks long). Are there any times when a student has caught up on sleep? Explain.

Solution: The total amount of sleep lost is $S = \int_0^{14} 7r(t)\,dt$. Using a calculator, we find $S \approx -6.44$, or about six and a half missed hours. A student has caught up on sleep at any time when the total amount of sleep lost is zero. Inspecting a graph of $r(t)$ and looking for where the areas above and below the t-axis are equal, we see that, for example, $\int_0^{4.5} r(t)\,dt \approx 0$ (so at $t \approx 4.5$ the sleep lost is 0), and similarly for $t \approx 7.75$, $t \approx 11$, etc.

3. (3 points) A math professor, consigned to endless purgatory for manifold grading sins, is grading an infinite stack of calculus quizzes. His grading rate r (in quizzes/day) increases as shown in the following table. In addition, his efficiency in red-ink usage u (in quizzes/gallon) depends on his quiz grading rate, as shown. Estimate the total amount of ink he uses for the time interval shown.

t (days)	0	15	30	45	60
r (quizzes/day)	25	30	37	45	53
u (quizzes/gallon)	20	22	24	26	30

Solution: Note that the number of gallons of ink used per quiz, and therefore the number of gallons per day, is given by

t (days)	0	15	30	45	60
r (quizzes/day)	25	30	37	45	53
u (quizzes/gallon)	20	22	24	26	30
$1/u$ (gallons/quiz)	1/20	1/22	1/24	1/26	1/30
r/u (gallons/day)	25/20	30/22	37/24	45/26	53/30

Therefore, we can estimate the total ink usage with either a left- or a right- hand sum (or, even better, the average of the two):

$$\text{total (left)} \approx 15\left(\frac{25}{20} + \frac{30}{22} + \frac{37}{24} + \frac{45}{26}\right) \approx 88.29 \text{ gallons}$$

$$\text{total (right)} \approx 15\left(\frac{30}{22} + \frac{37}{24} + \frac{45}{26} + \frac{53}{30}\right) \approx 96.04 \text{ gallons}$$

Our best estimate for the number of gallons of ink used is the average of these, or 92.17 gallons.