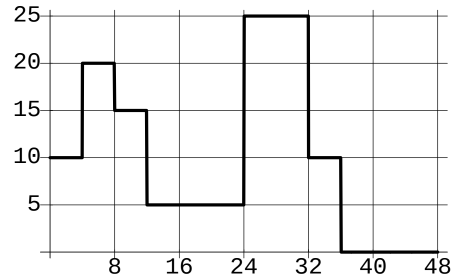
MATH 116-009 QUIZ 1 / 15 Sep 2006 Name:______________________________

1. A Calculus SWAT team is brought in to solve a particularly intractible problem. The graph to the right shows the number of Calculus Experts working on a two-day problem, as a function of the number of hours since the 8AM beginning of the first work day. They are paid \$50/hour for hours in a regular 8 hour work day (8AM–4PM), and \$75/hour for overtime (all other hours). How much does it cost to solve this Calculus problem? Be sure it is clear how and why you obtain the answer you do. (3 points)



Solution: Let $n(t)$ be the number of Calculus SWAT team members working at any time t, as given in the graph. The pay rate that we are given is in dollars per hour per person, so to get the rate in dollars per hour we need to multiply by $n(t)$. Thus for regular work hours (hours 0–8 and 24–32), the rate is $\$50 \cdot n(t)$/hour, and for overtime (hours 8–24 and 32–48) it is $\$75 \cdot n(t)$/hour. By the Fundamental Theorem of Calculus, the total amount spent is therefore

$$\int_0^8 50 \cdot n(t)\,dt + \int_8^{24} 75 \cdot n(t)\,dt + \int_{24}^{32} 50 \cdot n(t)\,dt + \int_{32}^{48} 75 \cdot n(t)\,dt$$
$$= 50\int_0^8 n(t)\,dt + 75\int_8^{24} n(t)\,dt + 50\int_{24}^{32} n(t)\,dt + 75\int_{32}^{48} n(t)\,dt$$

dollars. We can find the values of each of these integrals by calculating the area under $n(t)$ in the graph, finding total cost $= 50(40+80) + 75(60+60) + 50(200) + 75(40) = \$28,000$.

2. Suppose that the average American's salary increases at a rate of $r(t) = \$40(1.002)^t$ dollars/month, with t measured in months. What is the average rate of increase in the average American's salary over one year? (3 points)

Solution: The average rate of increase over a year (12 months) is $\frac{1}{12}\int_0^{12} r(t)\,dt = \frac{1}{12}\int_0^{12} 40(1.002)^t\,dt$ (dollars/month). We can estimate this numerically using a calculator, finding that the average rate of increase is approximately \$40.48/month.

3. A continuous function $f'(x)$ makes a wrong turn and ends up in the last question of a quiz, where a local calculus student notes that it has the following values.

$x =$	0	2	4	6	8	10
$f'(x) =$	0	-2	-4	0	16	24

Sketch a graph of the corresponding function $f(x)$, if you know that $f(4) = 8$. (4 points)

Solution: We know that the area under $f'(x)$ gives the change in the function value. Using a trapezoid, we can estimate that $\int_4^6 f'(x)\,dx \approx \frac{(2)(-4)+(2)(0)}{2} = -4$. Thus $f(6) \approx f(4) - 4 = 8 - 4 = 4$, and so on: $f(8) \approx 4 + \frac{(2)(0+16)}{2} = 20$, etc. This gives the values for $f(x)$ shown below. Using these we can generate the graph shown to the right.

$x =$	0	2	4	6	8	10
$f(x) \approx$	16	14	8	4	20	60