MATH 116-009 QUIZ 2 / 22 Sep 2006 Name:________________________________

1. An enterprising calculus student determines that for a particularly interesting (it is, after all, on a quiz) function $g(x)$, $\int_1^5 g(x)\,dx = 3$, $g(1) = 1$, and $g(5) = 2$. If possible, find $\int_1^5 x\,g(x^2+4)\,dx$ (if it isn't possible, explain why). (3 points)

Solution: We use substitution, letting $w = x^2 + 4$. Then $\frac{1}{2}dw = x$, and, noting that $w(1) = 5$ and $w(5) = 29$, we have $\int_0^1 x\,g(x^2+4)\,dx = \frac{1}{2}\int_5^2 9\,g(w)\,dw$. Which is problematic, because we don't know what the integral of g is for the domain $[5, 29]$. So we aren't able to find the value of this integral without further information.

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2. For the same tremendously interesting function $g(x)$ given in problem (1), find, if possible, $\int_1^5 x\,g'(x)\,dx$ (if it isn't possible, explain why). (3 points)

Solution: Here, we can use integration by parts. With $u = x$ and $v' = g'(x)$, $u' = 1$ and $v = g(x)$, so $\int_1^5 x\,g(x)\,dx = x \cdot g(x)\Big|_1^5 - \int_1^5 g(x)\,dx$. Plugging in the values we know, this is $\int_1^5 x\,g(x)\,dx = 5(2) - 1 - 3 = 6$.

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3. Let the function $f(x) = \int_0^{\cos(x)} \sin(t^2)\,dt + 2$. What is the slope of the tangent line to $f(x)$ at $x = \frac{\pi}{4}$? Find an estimate for the value of $f(\frac{\pi}{4})$ and use this to write an equation for the tangent line. (4 points)

Solution: The slope of the tangent line is $f'(\frac{\pi}{4})$. By the chain rule, we know that $f'(x) = -\sin(x) \cdot \sin(\cos^2(x))$, so $f'(\frac{\pi}{4}) = -\sin(\frac{\pi}{4}) \cdot \sin(\cos^2(\frac{\pi}{4})) = -\frac{1}{\sqrt{2}} \cdot \sin(\frac{1}{2})$. We can find the value of $f(\frac{\pi}{4})$ by numerically integrating $\int_0^{\cos(\pi/4)} \sin(t^2)\,dt$ to find $f(\frac{\pi}{4}) \approx 2.12$. Thus the tangent line is $y \approx 2.12 - \frac{\sin(\frac{1}{2})}{\sqrt{2}}(x - \frac{\pi}{4})$.