MATH 116-009 QUIZ 3 / 29 Sep 2006 Name:______________________________

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1. Find $\int \frac{x}{x^2-9}\,dx$ two ways. Neither of them should involve your calculator. For what it's worth, neither of them should involve integration by parts, either. (4 points)

Solution: We can solve this using substitution or by using partial fractions. With substitution, we take $w = x^2 - 9$. Then $\frac{1}{2}dw = xdx$, so that $\int \frac{x}{x^2-9}\,dx = \frac{1}{2}\int \frac{1}{w}\,dw = \frac{1}{2}\ln|x^2-9| + C$. A second way is to use partial fractions. Writing $\frac{x}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$, we have $x = A(x-3) + B(x+3)$. With $x = 3$, $B = \frac{1}{2}$, and with $x = -3$, $A = \frac{1}{2}$. Thus $\int \frac{x}{x^2-9}\,dx = \frac{1}{2}\int \frac{1}{x-3} + \frac{1}{x+3}\,dx = \frac{1}{2}(\ln|x-3| + \ln|x+3|)$. As a side note, these two solutions are the same because of the rule of logarithms, $\ln A + \ln B = \ln(A \cdot B)$.

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2. An astute calculus student notes that her calculus professor spews forth $r(t) = 5e^{7t^2}$ words/minute, where t gives the number of hours from the beginning of class. Use MID(3) to estimate the total number of words that the professor spews in the course of a standard 90 minute calculus II course. (3 points)

Solution: We note that 90 minutes is 1.5 hours, so to have three intervals our $\Delta t = 0.5$ hours. Then MID(3) estimates the integral $\int_0^{1.5} r(t)\,dt$ by finding the area under three rectangles whose heights are calculated from the midpoint of resulting three subdivisions of the interval. Thus $\text{MID}(3) = 0.5(r(0.25) + r(0.75) + r(1.25)) = 140,748$ words.
This is a little confusing, because $r(t)$ is given in words/minute, while Δt is in hours. This means that the calculation above isn't quite right—we're taking $(0.5\ \text{hours})(r(t)\ \text{words/min})$, and so would really have to also multiply by 60 min/hr to make the units work. For the purposes of this problem we didn't worry about that, however.

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3. Suppose that the exact value for the number of words the calculus professor in (2) spews is 1,706,082. What number of words would you expect to get if you used MID(12) to estimate the total? (No, you should not actually find MID(12).) (3 points)

Solution: The error in MID(3) is $1,706,082 - 140,748 = 1,565,334$ words. Going from MID(3) to MID(12) is four times as many steps, so we expect the error to drop by a factor of $4^2 = 16$, to $1,565,334/16 \approx 97,833$ words. Thus we expect MID(12) to estimate the number of words to be $1,706,082 - 97,833 = 1,608,249$ words.