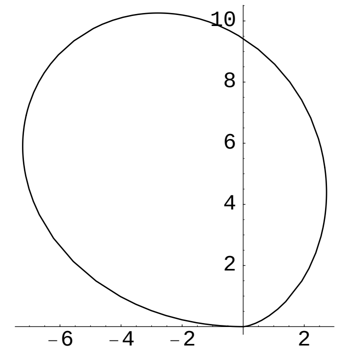
MATH 116-009 QUIZ 6 / 27 Oct 2006 Name:______________________________

1. Supposed that a child's throwing toy is (essentially) a two-dimensional piece of plastic lying within $r = 6\theta\sin(\theta)$ (where r is measured in inches), for $0 \leq \theta \leq \pi$. Find the surface area of the toy (the use of numerical integration is fine). (3 points)

Solution: The region is shown in the figure to the right. The area is given by the polar integral $\int_0^\pi \frac{1}{2}(36)\theta^2\sin^2(\theta)\,d\theta$, which we can find numerically to be $\int_0^\pi 18\theta^2\sin^2(\theta)\,d\theta \approx 78.88$ in^2 (actually $3\pi^3 - \frac{9}{2}\pi$).



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2. Now, suppose that the density of the toy is given to be $\delta(\theta) = 1 + \frac{\theta}{2}$ g/in^2. Find the mass of the toy. (Again, using numerical integration is fine.) (3 points)

Solution: A slice of area from the toy is $\Delta A = \frac{1}{2}(36)\theta^2\sin^2(\theta)\,\Delta\theta$ in^2, so the mass of this slice is $\Delta M = (1+\frac{\theta}{2})(\frac{1}{2})(36)\theta^2\sin^2(\theta)\,\Delta\theta$ g. The total mass is thus $\int_0^\pi (1+\frac{\theta}{2})(18\theta^2\sin^2(\theta))\,d\theta$. Again, integrating numerically, we find $M \approx 155.2$ g.

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3. Set up but do not evaluate an expression to find the y-center of mass of a 2 inch wide by 3 inch high rectangular object having a density $\delta(y) = \cos(y)$ g/in^2. (y is the vertical coordinate, which measures the height of the object.) (4 points)

Solution: We know that the center of mass is given by

$$\overline{y} = \frac{\int_0^3 2y\cos(y)\,dy}{\int_0^3 2\cos(y)\,dy}.$$