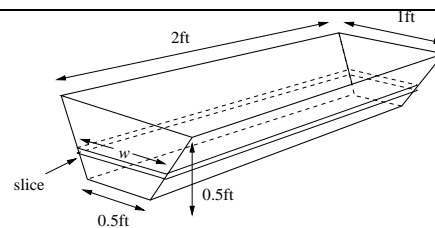


1. An exceptionally thirsty polar weasel finds a convenient water basin, fed by a pure artesian spring pulling uncontaminated water from glacier depths. Suppose the basin, shown to the right, contains water 0.3ft deep when the weasel plugs the basin so that it does not refill. If the weasel then drinks the basin dry, how much work does it do in moving all of the water to the top of the basin? (The weight of water is 62.4 lb/ft<sup>3</sup>.) (4 points)

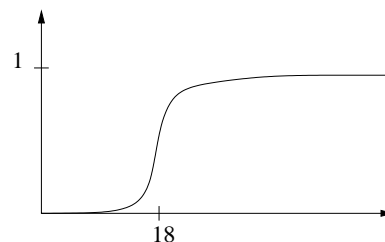


*Solution:* Let  $h$  be measured up from the bottom of the basin. Then the work to lift a horizontal “slice” of water at a height  $h$  out of the basin is  $W_s = (62.4)(A_s)\Delta h \cdot (0.5 - h)$ , where  $A_s$  is the horizontal cross-sectional area of the “slice.”  $A_s = w \cdot 2$ , where  $w$  is the width of the slice along the trapezoidal face of the basin, as shown in the figure. Because the width is linear with  $h$ , and  $w(0) = 0.5$  while  $w(0.5) = 1$ ,  $w = 0.5 + h$ . Thus the total work is

$$\int_0^{0.3} 62.4 \cdot 2(h + 0.5) \cdot (0.5 - h) dh = \int_0^{0.3} 124.8 \cdot (0.25 - h^2) dh = 124.8 \left( 0.25h - \frac{1}{3}h^3 \right) \Big|_0^{0.3} \approx 8.24 \text{ ft} \cdot \text{lb}.$$

2. Let  $g(t)$  be a function with the property that  $g(T)$  gives the fraction of students in math 116 in the fall term who are no older than  $T$  years. Sketch a reasonable graph of  $g(t)$ . Is it a density function? (3 points)

*Solution:* A reasonable sketch is shown to the right. Most students in math 116 in the fall are first-year students, so there is a large increase in the fraction of students at age 18. This is a cumulative density function.



3. Let  $g(t)$  be the function in problem (2). What is the meaning of  $\int_{17}^{19} g'(t) dt$ ? (3 points)

*Solution:*  $\int_{17}^{19} g'(t) dt = g(19) - g(17)$ . This is the fraction of students between the ages of 17 and 19. We can see this from the definition of the CDF ( $g(19)$  = the fraction of students less than 19, and similarly for  $g(17)$ ), so that  $g(19) - g(17)$  is those students less than 19 but not less than 17), and from the fact that  $\int_{17}^{19} g'(t) dt$  is just the integral of a probability density function between the values 17 and 19.