MATH 116-009 QUIZ 8 / 10 Nov 2006 Name:________________________________

1. Suppose that the number of polar weasels in southeast Michigan is increasing at a rate of 4% per year, and that at the last census (in 2000) the population of weasels was 3142. Let p_n be the population of weasels n years after 2000. (3 points)

a. Find a formula for p_n.

b. Does p_n converge as $n \to \infty$?

Solution: We know that $p_0 = 3142$. Then $p_1 = (1.04)(3142) = 3268$, $p_2 = (1.04)p_1 = (1.04)^2(3142) = 3398$, and so on. Thus $p_n = 3142 \cdot (1.04)^n$, which diverges to ∞ as $n \to \infty$.

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2. The polar weasels, concerned about loss of natural habitat, have opened a space station in hopes of colonizing space. On the first and every successive month following completion of the space station, a space capsule piloted by skilled astro-weasel-nauts arrives and releases 12 ft^3 of excess carbon dioxide into the space station. The air filtration systems on the station can remove 95% of excess carbon dioxide in a month. Let C_n be the amount of excess CO_2 in the station at the end of n months. Find a closed-form expression for C_n. (3 points)

Solution: At the end of one month there is an infusion of 12 ft^3 of CO_2 into the station, so that $C_1 = 12$. Then at the end of the second month 5% of this remains, and there is another infusion of 12 ft^3 of CO_2, so $C_2 = (0.05)(12) + 12$. Similarly, $C_3 = (0.05)^2(12) + (0.05)(12) + 12$, and so on. Thus $C_n = (0.05)^{n-1}(12) + (0.05)^{n-2}(12) + \cdots + (0.05)(12) + 12$. This is a finite geometric series with n terms, so $C_n = 12\left(\frac{1-0.05^n}{1-0.05}\right) = 12.63(1 - 0.05^n)$.

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3. Which, if any, of the following series converge? (4 points)

a. $\Sigma \frac{n+1}{n^2+2n+1}$ **b.** $\Sigma \frac{e^n}{e^n+5}$

Solution: **a.** We can easily use the integral test here: the terms in the series, $\frac{n+1}{n^2+2n+1}$, are positive and decreasing, so the convergence of $\Sigma \frac{n+1}{n^2+2n+1}$ will be the same as that of $\int_1^\infty \frac{n+1}{n^2+2n+1}\,dn = \int_1^\infty \frac{1}{n+1}\,dn$. $\int_1^\infty \frac{1}{n+1}\,dn = \lim_{b\to\infty} \ln(b+1) - \ln(2)$, which diverges. Thus $\Sigma \frac{n+1}{n^2+2n+1}$ must also diverge.

b. We note that the terms in this sequence, $\frac{e^n}{e^n+5}$, do not go to zero as $n \to \infty$: $\lim_{n\to\infty} \frac{e^n}{e^n+5} = 1$. Thus this series must also diverge.