

*It may or may not be useful to note that:*

$$\begin{aligned} \sin x &= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \dots \\ e^x &= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots \end{aligned}$$

$$\begin{aligned} \cos x &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots \\ (1+x)^p &= 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \dots \end{aligned}$$

1. What is the radius of convergence of  $\sum_{n=0}^{\infty} \frac{3^n x^n}{n+2}$ ? (3 points)

2. Suppose that the Taylor series for a function  $f(x)$  is given to be  $f(x) = 2x + \frac{8x^3}{2!} + \frac{32x^5}{4!} + \frac{128x^7}{6!} + \dots$ . What are  $f(0)$ ?  $f'''(0)$ ?  $f^{(19)}(0)$ ? (3 points)

3. A wandering polar weasel meditates for 2.718 minutes and then sketches the graph to the right, which shows three functions for values of  $x$  near 0. Astonishingly, one of these turns out to be exactly  $\frac{1}{1-x^2}$ , one  $2 - \cos(x)$ , while the third is another function that remains anonymous to protect its identity. Which of the graphs correspond to each of the two functions specified? (4 points)

