MATH 116-009 QUIZ 10 / 8 Dec 2006 Name:________________________

1. Consider the differential equation $y'' + ky = 0$, where k is a constant. For what values of k, if any, is $y = e^{3x}$ a solution? (3 points)

Solution: If $y = e^{3x}$, then $y'' = 9e^{3x}$. Plugging in, we have $9e^{3x} + ke^{3x} = 0$. Thus $k = -9$.

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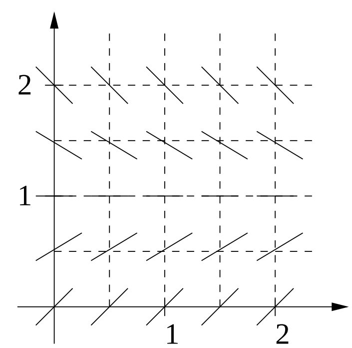
2. Consider the slope field shown to the right. Let $y = f(x)$ be a solution to the corresponding differential equation that passes through the point (0,2).
(a) Can $f(x) < 1$? Explain.
(b) If we use Euler's method with $\Delta x = 1.5$, can our approximation to y be less than 1? Explain.
(4 points)



Solution: **(a)** We can never have $y = f(x) < 1$. This is because the slope field indicates that when $y = 1$ the slope is zero. Thus if $f(x)$ decreases from its initial value of 2 to 1 it will then have the value 1 for all larger values of x.
(b) However, we would expect that Euler's method with $\Delta x = 1.5$ would give values less than 1. At (0,2) the slope appears to be approximately -1. Thus if $\Delta x = 1.5$, we would approximate $y(1.5) \approx 2 + (1.5)(-1) = 0.5$. Euler's method uses the same slope for the entire distance Δx and thus, in this case, overshoots the actual value of y.

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3. Solve $tz\frac{dz}{dt} = 3t^2 + 4$. (3 points)

Solution: Separating variables, we have $z\,dz = (3t + \frac{4}{t})\,dt$. Integrating gives $\frac{1}{2}\,z^2 = \frac{3}{2}\,t^2 + 4\ln|t| + C$. Thus $z = \pm\sqrt{3\,t^2 + 8\ln|t| + C}$.