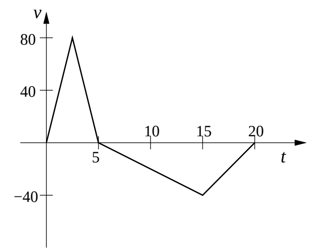
MATH 116-023 QUIZ 1 / 12 Jan 2006 Name:____________________

1. If the velocity of a fast-moving overly-verbose coffee-imbibing calculus professor is given (in ft/min) by the graph below, how far does the professor travel in the 20 min interval shown? When is the professor farthest from his starting point? (4 points)



Solution: Distance traveled is the area "under" the graph of the velocity. If we assume that positive velocities indicate motion to the right, then the professor moves 200 ft ($= \frac{1}{2}(5)(80)$) to the right for $0 \le t \le 5$, and 300 ft ($= \frac{1}{2}(10)(40) + \frac{1}{2}(5)(40)$) to the left for $5 \le t \le 20$. Thus the professor is farthest from his starting position when $t = 5$ min (200 ft to the right) and ends up, at $t = 20$, 100 ft to the left of the starting position.

2. Suppose that an alert calculus student estimates, for some $f(x)$, $\int_0^3 f(x)\,dx$, by using a left-hand sum and also by using a right-hand sum with the same number of intervals. She finds $\int_0^3 f(x)\,dx \approx 25.5$ with the left-hand sum and $\int_0^3 f(x)\,dx \approx 28.5$ with the right-hand sum. If $f(x)$ increases from $f(0) = 6$ to $f(3) = 12$, how many intervals did the student use for the two sums? (3 points)

Solution: The difference between the left- and right-hand sum is $\text{RHS} - \text{LHS} = \Delta x(f(3) - f(0))$, so $28.5 - 25.5 = \Delta x(12 - 6)$, and $\Delta x = \frac{1}{2}$. Thus she must have used $n = 6$ intervals.

3. It is well documented that students' love of calculus increases as time passes. If the rate (given in joy-filled moments per week) at which this love increases is shown in the following table, **(a)** express the number of joy-filled moments a calculus student may be expected to experience in the six-week interval shown as a definite integral, and **(b)** estimate this number. Be sure it is clear how you obtain your estimate. (3 points)

t (weeks)	0	2	4	6
$r(t)$ (jfm/wk)	3	9	17	32

Solution: The number of joy-filled moments is given by $\int_0^6 r(t)\,dt$, which we can estimate with a Riemann sum. A left-hand sum gives $\int_0^6 r(t)\,dt \approx 2(3 + 9 + 17) = 58$ joy-filled moments, and a right-hand sum gives $\int_0^6 r(t)\,dt \approx 2(9 + 17 + 32) = 116$ joy-filled moments. A reasonable estimate may therefore be the average of these, 87 joy-filled moments.