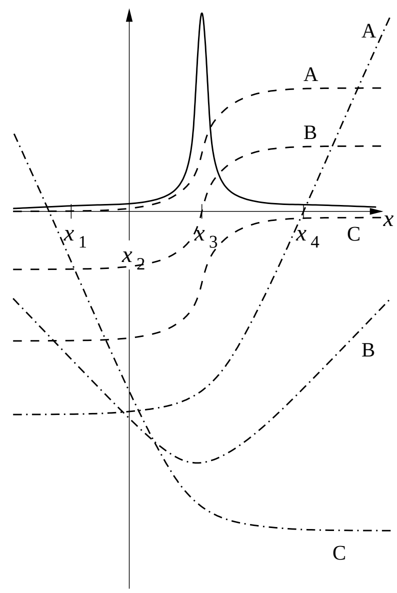
MATH 116-023 QUIZ 2 / 19 Jan 2006 Name:____________________________

1. Suppose that $f(x)$ is an odd function and that $\int_{-2}^{5} 3f(x) + 2\,dx = 23$. **(a)** What is $\int_{2}^{5} f(x)\,dx$? **(b)** A passing calculus fan asserts that $f(x) \geq 1$ for $2 \leq x \leq 5$; given the information in this problem, is this assertion correct? (4 points)

Solution: **(a)** We know that $\int_{-2}^{5} 3f(x) + 2\,dx = 3\int_{-2}^{5} f(x)\,dx + \int_{-2}^{5} 2\,dx = 3\int_{-2}^{5} f(x)\,dx + 14$, so $3\int_{-2}^{5} f(x)\,dx = 9$. Then, because $f(x)$ is odd, we know that $\int_{-2}^{2} f(x)\,dx = 0$, so $3\int_{2}^{5} f(x)\,dx = 9$, and $\int_{2}^{5} f(x)\,dx = 3$. **(b)** The assertion is probably not correct. If $f(x) = 1$ for $2 \leq x \leq 5$ then certainly $\int_{2}^{5} f(x)\,dx = 3$. However, if $f(x)$ is not constant for this range of x-values, it must take on some values larger (and therefore also smaller) than one for the area to equal exactly three.

2. Suppose that $f''(x)$ is graphed in the figure to the right. Sketch graphs of $f'(x)$ and $f(x)$, indicating on your graphs the locations of the points x_1, x_2, x_3 and x_4. (3 points)

Solution: The graph of $f''(x)$ (the solid line), $f'(x)$ (dashed), and $f(x)$ (dash-dotted) are shown in the figure. Note that the vertical location of $f'(x)$ is arbitrary; three different possibilities, marked "A", "B" and "C", are shown here. The behavior of $f(x)$ *does*, however, depend on the location of $f'(x)$; the corresponding graphs of $f(x)$ for the three possible $f'(x)$ graphs are shown in the figure. Of course, the vertical location of $f(x)$ is again arbitrary.



3. Find each of the following: (3 points)

(a) $\int 3x^3 - 4\sqrt[3]{x}\,dx$ **(b)** $\int \sin(2y) - \frac{1}{\cos^2(y)}\,dy$ **(c)** $\int \frac{(z-1)^2}{z^2}\,dz$

Solution:
(a) $\int 3x^3 - 4\sqrt[3]{x}\,dx = \frac{3}{4}x^4 - 3x^{4/3} + C$.
(b) $\int \sin(2y) - \frac{1}{\cos^2(y)}\,dy = -\frac{1}{2}\cos(2y) - \tan(y) + C$.
(c) $\int \frac{(z-1)^2}{z^2}\,dz = \int \frac{z^2-2z+1}{z^2}\,dz = \int 1 - 2z^{-1} + z^{-2}\,dz = z - 2\ln(|z|) - z^{-1} + C$.