MATH 116-023 QUIZ 6 / 21 Feb 2006 Name:______________________________

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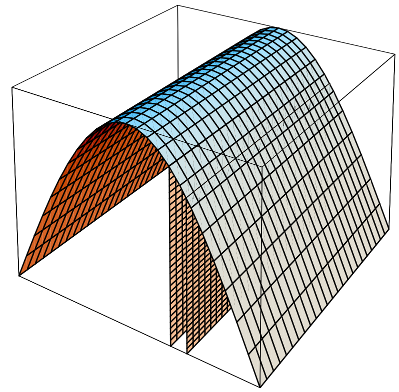
1. A long shed is created by constructing a front frame with the shape $y = H\sin(x)$, for $0 \le x \le \pi$ m, and extending this shape with a length L. Sketch the shed and a representative slice that you could use to find the volume by integration. Set up the integral and find the volume of the shed. (3 points)

Solution: The shed is shown in the figure to the right, along with a representative slice taken a distance x from the left edge of the shed. The volume of the slice is $V_{sl} = L \cdot H\sin(x) \cdot \Delta x$, so that the total volume of the shed is given by $\int_0^{\pi} L \cdot H\sin(x)\,dx = 2LH$ m^3.



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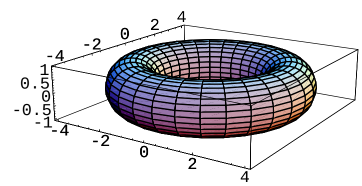
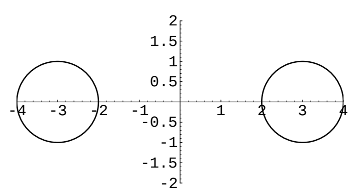
2. One model for the shape of a space station is a donut shape that spins, so that in the ring there is a perceived "gravity" pulling outwards. Suppose that such a space station is given by the graph of $(x-3)^2 + y^2 = 1$ (where all units are, of course, "space station length units," sslus), rotated around the y-axis. Set up an integral to find the volume enclosed by such a space station. (3 points)

Solution: The "space station" and its xy-cross section are shown in the figures to the right. If we cut the volume up with horizontal slices, then we get washers with an inside radius of $x = 3 - \sqrt{1-y^2}$ and outside radius $x = 3 + \sqrt{1-y^2}$. Thus the volume of the station is

$$V = \int_{-1}^{1} \pi(3+\sqrt{1-y^2})^2 - \pi(3-\sqrt{1-y^2})^2\,dy$$
$$= \int_{-1}^{1} 12\pi\sqrt{1-y^2}\,dy.$$

We could evaluate this by trigonometric substitution or numerically, to get $V = 12\pi^2$ (≈ 118.44) sslu^3.





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3. Sketch the graphs of the curves given in polar coordinates by $r = 1 + \sin(\theta)$ and $r = \sin(\theta)$. If we want to find the area inside $r = 1 + \sin(\theta)$ but outside of $r = \sin(\theta)$, sketch an appropriate "slice." Set up an integral to find this area. (3 points)

Solution: The two curves are shown to the right. The slice has angular width $\Delta\theta$. The area inside the inner circle is therefore given by $A_{ci} = \int_0^{\pi} \frac{1}{2}\sin^2\theta\,d\theta = \frac{\pi}{4}$ (which makes pretty good sense, given that it's a circle of radius $\frac{1}{2}$). The area inside the cardioid is $A_{ca} = \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2\,d\theta = \frac{3\pi}{2}$, so the area between the two is $A = \frac{3\pi}{2} - \frac{\pi}{4} = \frac{5\pi}{4}$. Of course, if we were just setting up the integral, we could stop with saying $A = \int_0^{\pi} \frac{1}{2}\sin^2\theta\,d\theta - \int_0^{2\pi} \frac{1}{2}(1+\sin\theta)^2\,d\theta$.