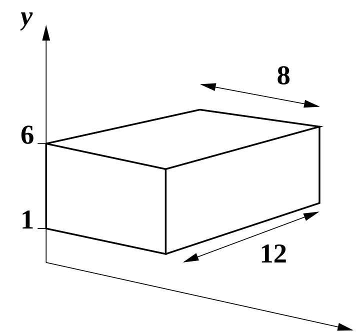
MATH 116-023 QUIZ 7 / 9 Mar 2006 Name:____________________

1. A somewhat questionable model for the mass distribution of a truck or SUV is the following: the SUV is a rectangular solid 8 ft wide by 5 ft tall by 12 ft long, 1 ft above the ground (because of its wheels, of course—note that this essentially says that the SUV extends from the ground to a height of 6 ft, but has zero mass for the lowest 1 ft). This is shown in the figure to the right. Suppose that the density of the truck is approximately $\delta(y) = \frac{20}{3}(6-y)$ lbs/ft^3, where y is the distance up from the ground. If the weight of the truck is 8000 lbs, find its y-center of mass. (4 points)



Solution: The y-center of mass is moment $\int_1^6 y \cdot \delta(y) \cdot a(y)\, dy$, where $a(y)$ is the area of a horizontal cross-section of the truck, divided by the mass (weight). Thus

$$\begin{aligned} \overline{y} &= \frac{\int_1^6 y \cdot (\frac{20}{3}(6-y)) \cdot 8 \cdot 12\, dy}{8000} \\ &= \frac{640}{8000} \int_1^6 6y - y^2\, dy = \frac{8}{3} \quad \text{ft.} \end{aligned}$$

2. Find the work required to empty a cylindrical tank, standing on one of its circular ends, with radius $r = 2$ m and height $h = 4$ m if it is initially half full of water (mass 1000 kg/m^3; use $g = 9.8$ m/s^2). (4 points)

Solution: We slice the tank horizontally into slices of thickness Δy, where y is the height measured from the bottom of the tank. Then the weight of a slice is $w_{sl} = (\pi r^2 \Delta y)(9800) = 39,200\pi\Delta y$. The slice has to be lifted a height $h = 4 - y$, so the total work is $\int_0^2 39,200\pi(4-y)\, dy = 235,200\pi$ J. Or, approximately, 738,903 J.

3. True or false (explain in one sentence): If $f(t)$ is a density function such that $f(t)\Delta t$ gives the fraction of the U.S. population having taken between t and $t + \Delta t$ years of math classes, then $\int_{13}^{\infty} f(t)\, dt \geq 0.50$. (2 points)

Solution: This is probably false. $\int_{13}^{\infty} f(t)\, dt$ gives the fraction of the population that has had in excess of 13 years of mathematics classes, so for it to be greater than 0.5 would require that over 50% of the entire population to have completed college-level mathematics. . . assuming that they had math for 12 years in K-12. Even if a majority of the U.S. population goes to college, it seems unlikely that the fraction of this population that takes math will add up to 50% of the entire U.S.