MATH 116-023 QUIZ 8 / 16 Jan 2006 Name:________________________________

1. Give an explicit formula for a sequence s_n which has properties that $s_1 = 2$, $\lim_{n\to\infty} s_n = 1$, and $s_n < 1$ for some values of n. What are the first four terms in your sequence? (4 points)

Solution: Answers will vary. One such sequence is $s_n = 1 - \frac{(-1)^n}{n}$, for which the first four terms are $s_1 = 2$, $s_2 = \frac{1}{2}$, $s_3 = \frac{4}{3}$ and $s_4 = \frac{3}{4}$.

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2. A daring calculus-loving student leaps from a tree-house located 15 feet above the surface of a trampoline. She then bounces on the trampoline 10 times, attaining a height after each bounce that is $\frac{1}{2}$ her previous height. **(a)** write a series giving the total vertical distance she travels after the first bounce from the trampoline (assume that she comes to a stop upon landing on the trampoline for the 10th time), and **(b)** determine its sum. (4 points)

Solution: After the first bounce she will attain a height of $(15)(\frac{1}{2}) = 7.5$ ft, and then will fall an equal distance back to the trampoline, traveling a total distance of 15 ft by the time she lands on the trampoline the second time. After the second bounce she will travel an additional $2(15)(\frac{1}{2})^2 = 7.5$ $(= 15(\frac{1}{2}))$ ft before landing the third time, and so on (before landing the fourth time, she will travel an additional $15(\frac{1}{2})^2$ ft, etc.). Thus the total distance she travels is $D = \sum_{n=0}^{8} 15(\frac{1}{2})^n$. This is a finite geometric series with nine terms, so the sum is $D = 15\frac{1-(\frac{1}{2})^9}{1-\frac{1}{2}} \approx 29.94$ feet.

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3. Give an integral that could be used to test the convergence of $\sum_{n=0}^{\infty} (n-2)e^{-(n-2)}$. Without evaluating the integral, how would it tell you if the series converges or not? (3 points)

Solution: The series is $-2e^2 - e + 0 + e^{-1} + 2e^{-2} + 3e^{-3} + \cdots$, which has positive decreasing terms for $n > 3$. We can thus determine the convergence of the series by considering $\int_3^\infty (n-2)e^{-(n-2)}\,dn$. If we were to evaluate the integral (which we could do by substituting $w = n - 2$, to get $\int_1^\infty we^{-w}\,dw$, which we would then integrate by parts; the value of the integral is $2e^{-1}$), we would know that the series converged if the integral converges (which it does). Similarly, if the integral diverged, so would the series.