

1. Find the radius of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(n!)^2(x-3)^n}{(2n+1)!}$ . What are the endpoints of the interval of convergence? How would you know if the series converged at the endpoints (do not actually test this)? (4 points)

*Solution:* To find the radius of convergence, we use the ratio test:

$$\begin{aligned} \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} &= \lim_{n \rightarrow \infty} \left| \frac{(n+1)!(n+1)(x-3)^{n+1}}{(2n+3)!} \right| \cdot \left| \frac{(2n+1)!}{n!n!(x-3)^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{(n+1)(n+1)(x-3)}{(2n+2)(2n+3)} \right| = \frac{|x-3|}{4}. \end{aligned}$$

Thus the radius of convergence is 4, and the endpoints of the interval of convergence are  $-1$  and  $7$ . To test convergence at these points, we would plug them into the series and use some other convergence tests (e.g., the comparison test).

2. Suppose that the following table gives points on the function  $f(x)$ . If  $P_2 = a + bx + cx^2$  is the second degree Taylor polynomial for  $f(x)$  about  $x = 0$ , (a) are  $a$ ,  $b$  and  $c$  positive or negative?, and (b) what are (reasonable estimates for)  $a$  and  $b$ ? (3 points)

$x$	$-0.15$	$0$	$0.15$
$f(x)$	$3.125$	$3.25$	$3.55$

*Solution:* We note that  $f(0) = 3.25 > 0$ ,  $f(x)$  is increasing (so that  $f'(0) > 0$ ), and that the rate of increase is bigger between  $x = 0$  and  $x = 0.15$  than it is between  $x = -0.15$  and  $x = 0$  (so that  $f''(0) > 0$ ). Thus  $a > 0$ ,  $b > 0$  and  $c > 0$ . Then  $a = f(0) = 3.25$ , and  $b = f'(0) \approx \frac{3.55-3.125}{0.3} = 1.417$ .

3. Find an expression for the general term of the Taylor series  $\sin(x) \cos(x) = \frac{x}{2} - \frac{x^3}{2(3!)} + \frac{x^5}{2(5!)} - \frac{x^7}{2(7!)} + \dots$  and give the starting value of the index ( $n$ ). (3 points)

*Solution:*  $\sin(x) \cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2(2n+1)!}$ .