MATH 116-023 QUIZ 10 / 6 Apr 2006 Name:____________________________________

Possibly useful formulae (all series around $x = 0$): $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$; $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$; $\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$; $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$; and $(1+x)^p = 1 + px + \frac{p(p-1)}{2!}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots$.

1. Use Taylor series to determine which of $f(z) = \sin(z^2)$ or $g(z) = z(e^z - 1)$ is smaller near $z = 0$ (consider $z > 0$). (3 points)

2. Find a value of r for which $y = x^r$ is a solution to the differential equation $x^2 y'' - 2y = 0$. (3 points)

3. On the slope field for the differential equation $\frac{dy}{dx} = (y+1)\sin(x)$ shown below, **(a)** sketch the solution to the differential equation that passes through $(0, 0.25)$, and **(b)** sketch the Euler solution to this differential equation, starting at $y(0) = 0.25$ and using a step size of $\Delta x = 0.25$, that finds $y(1)$. *Note that you do not need to actually calculate the Euler method solution for (b)!* (4 points)