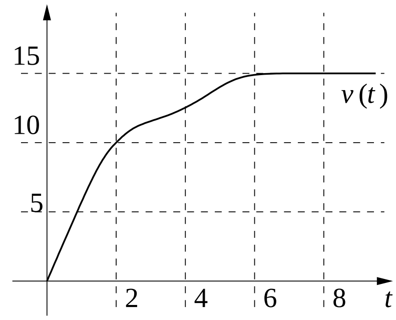
MATH 116-028 QUIZ 1 / 4 Jan 2007 Name:______________________________

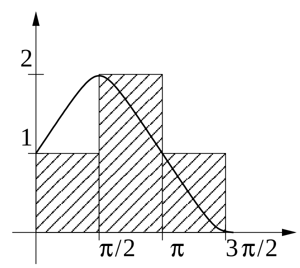
1. A calculus student is racing to get to the gateway lab to start taking the Entrance Gateway at the instant the doors open. The student's velocity, $v(t)$ (in m/s) is shown in the graph to the right for $0 \le t \le 8$ seconds. Write an integral that gives the distance the student travels in those 8 sec, and estimate this distance. (3 points)



Solution: The distance travelled is just $\int_0^8 v(t)\,dt$. We can approximate this with a left- or right- hand sum, or the average of the two. A left-hand sum is $\int_0^8 v(t)\,dt \approx 2(0+10+12.5+15) = 75$ m, and a right-hand sum $\int_0^8 v(t)\,dt \approx 2(10+12.5+15+15) = 105$ m. The average is likely to be a more accurate estimate, giving $\int_0^8 v(t)\,dt \approx$ 90 m.

2. Consider the integral $\int_0^{3\pi/2} 1+\sin(x)\,dx$. Let LHS($n$) and RHS($n$) be, respectively, the left- and right-hand sums with n subdivisions approximating this integral. *By looking at a graph(—not by evaluating them)*, place in increasing order the following quantities: LHS(3), RHS(1), and $\int_0^{3\pi/2} 1+\sin(x)\,dx$. (3 points)

Solution: See the figure to the right. Clearly LHS(3) > the area under the curve, and at $x = \frac{3\pi}{2}$, $1+\sin(3\pi/2) = 0$, so RHS(1) = 0. Thus we have RHS(1) $< \int_0^{3\pi/2} 1+\sin(x)\,dx <$ LHS(3).



3. Find each of the following derivatives (you need not simplify your answers). (4 points)

a. $\frac{d}{dx}(3x\sin(x^2+1))$

b. $\frac{d}{dt}(\frac{e^{2t}}{\ln(t)})$

Solution: **a.** $\frac{d}{dx}(3x\sin(x^2+1)) = 3\sin(x^2+1) + 6x^2\cos(x^2+1)$.

b. $\frac{d}{dt}(\frac{e^{2t}}{\ln(t)}) = \frac{2e^{2t}\ln(t) - \frac{1}{t}e^{2t}}{(\ln(t))^2}$ (or, $= 2e^{2t}(\ln(t))^{-1} - e^{2t}(\frac{1}{t})(\ln(t))^{-2}$).