MATH 116-028 QUIZ 2 / 15 Jan 2008 Name:_____

1. The following two statements are false. Explain, without any calculation, why they are false. (4 points)

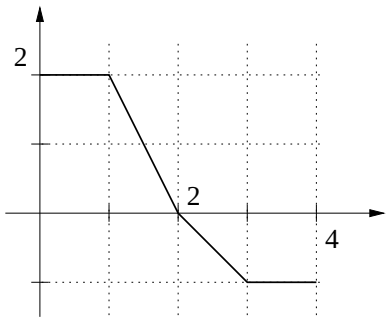
a. $\int_{-1}^{0} e^{-x^2}\,dx = -0.746824$

b. $\int_{-1}^{1} x\,e^{-x^2}\,dx = 0.632121$

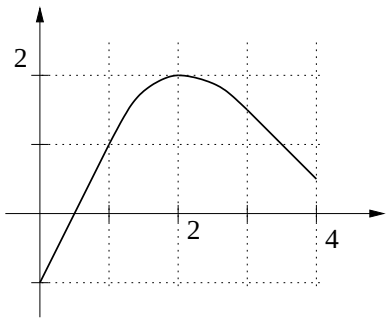
Solution: **a.** We know that e^{-x^2} is a positive function, so the area between the graph of the function and the x-axis must lie above the x-axis, and the integral must therefore be positive.

b. We know that $x\,e^{-x^2}$ is an odd function, so the integral from -1 to 1 must be zero.

2. Suppose that the function f is shown in the figure to the right. If $F' = f$ and $F(0) = -1$, carefully sketch a graph of $F(x)$ for $0 \le x \le 4$. (3 points)



Solution: We know that $F(0) = -1$, and that from $x = 0$ to $x = 1$ its slope is a constant (2). From $x = 1$ to $x = 2$, $F(x)$ must continue to increase from $(1, 1)$ to $(2, 2)$ (because the area under $f(x)$ is one), and its slope must decrease to zero. From $x = 2$ to $x = 3$ it decreases by one-half to $(3, \frac{3}{2})$ and the slope decreases from zero to -1. From $x = 3$ to $x = 4$ the slope is a constant -1. This is shown in the figure to the right, below.



3. If the average value of $f(x) = 9x^2$ on the interval $0 \le x \le b$ is 48, what is b? (3 points)

Solution: The average value we want is $\frac{1}{b-0}\int_0^b 9x^2\,dx = \frac{1}{b}(3x^3)\Big|_0^b = 3b^2$. Thus we need $3b^2 = 48$, so $b^2 = 16$ and $b = \pm 4$.