MATH 116-028 QUIZ 4 / 29 Jan 2008 Name:____________________________

1. Find, by hand: $\int \frac{e^x}{4e^{2x}-1}\,dx$. (3 points)

Solution: Note that we can factor the denominator of the integrand: $\int \frac{e^x}{4e^{2x}-1}\,dx = \int \frac{e^x}{(2e^x+1)(2e^x-1)}\,dx$. Thus substituting $w = e^x$ (so that $dw = e^x\,dx$) reduces this to a simple fraction:

$$\int \frac{e^x}{(2e^x+1)(2e^x-1)}\,dx = \int \frac{dw}{(2w+1)(2w-1)}.$$

We can simplify this with partial fractions. Taking $\frac{1}{(2w+1)(2w-1)} = \frac{A}{(2w+1)} + \frac{B}{(2w-1)}$, we have $1 = (2w-1)A + (2w+1)B$. Then, if $w = \frac{1}{2}$, $B = \frac{1}{2}$, and if $w = -\frac{1}{2}$, $A = -\frac{1}{2}$. Thus

$$\int \frac{dw}{(2w+1)(2w-1)} = -\frac{1}{2}\int \frac{dw}{2w+1} + \frac{1}{2}\int \frac{dw}{2w-1} = -\frac{1}{4}\ln|2w+1| + \frac{1}{4}\ln|2w-1| + C.$$

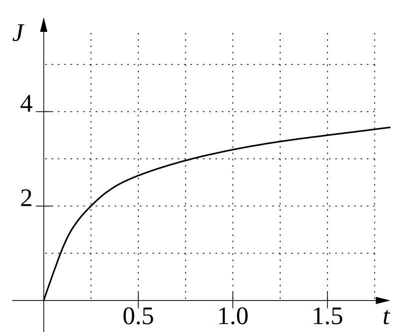
Back-substituting for w, we have $\int \frac{e^x}{4e^{2x}-1}\,dx = -\frac{1}{4}\ln(2e^x+1) + \frac{1}{4}\ln|2e^x-1| + C$. (Note that the absolute values are not necessary for the first term because $2e^x + 1 > 0$.)

2. For each of the following integrals, indicate which of **substitution**, **integration by parts**, **long division**, **partial fractions**, a **table** of integrals, or **no method** would be the logical *first step* toward finding the integral. Do not find any of these integrals. (3 points)

a. $\int \arctan(x)\,dx$ **b.** $\int \frac{1}{3+(2z+1)^2}\,dz$ **c.** $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

Solution: **(a.)**: We don't know how to antidifferentiate $\arctan(x)$, and it isn't on our table of integrals, but we do know how to differentiate it ($\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$), so the most logical choice is to use **integration by parts** with $u = \arctan(x)$ and $v' = 1$.
(b.): The most logical choice here is **substitution** with $w = 2z+1$ (or $w = \frac{2z+1}{\sqrt{3}}$), followed by use of a table (or recognition of the integral as an arctangent).
(c.): Here we should clearly use **substitution** with $w = \sqrt{x}$, because we then have $2\,dw = \frac{1}{\sqrt{x}}\,dx$, which is in the integral as well.

3. An enterprising calculus student, intrigued by the amount of fun that her calculus class is having, does a careful investigation of the joy, J, being experienced by the class during a typical class period. The graph gives J (in deleriums/hour, the usual unit) for the 1.5 hour class period. Estimate with MID(3) the total number of deleriums of joy experienced by the class. Is your estimate an over- or under-estimate? (4 points)



Solution: The total joy is given by $\int_0^{1.5} J(t)\,dt$, which we can estimate with MID(3) by reading values from the graph:

$$\int_0^{1.5} J(t)\,dt \approx (0.5)(2+3+3.5) = 4.25 \text{ deleriums}.$$

Because the graph is concave down we know that the midpoint approximation is an overestimate.