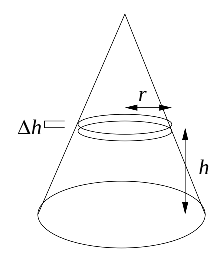
MATH 116-028 QUIZ 5 / 12 Jan 2008 Name:______________________

1. Find parametric equations for the line tangent to the curve given by $x(t) = 3t\cos(2t)$, $y(t) = 2\sin(2t)$ when $t = \frac{\pi}{2}$. (3 points)

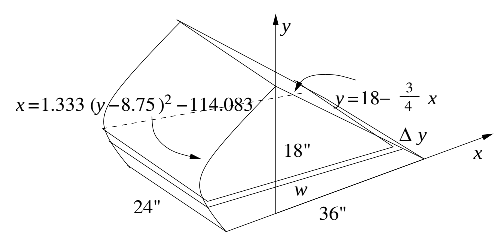
Solution: We note that $x'(t) = 3\cos(2t) - 6t\sin(2t)$, so $x'(\pi/2) = -3$, and $y'(t) = 4\cos(2t)$, so $y'(\pi/2) = -4$. At $t = \pi/2$, we pass through the point $(x(\pi/2), y(\pi/2) = (-\frac{3\pi}{2}, 0)$, so the tangent line is given by $x(t) = -\frac{3\pi}{2} - 3(t - \frac{\pi}{2}) = -3t$, $y(t) = -4(t - \frac{\pi}{2}) = -4t + 2\pi$.

2. The integral $\int_0^5 \pi(2 - \frac{2h}{5})^2\, dh$ represents the volume of either a hemisphere or a cone. Which is it? What are the dimension(s) of the volume? Sketch one slice that would be used to calculate the volume, showing how the dimension(s) are related to the slice. (3 points)

Solution: This is a cone, as shown to the right. Each slice is a disk with radius $r = 2 - \frac{2h}{5}$, for $0 \le h \le 5$ and height Δh, so that the volume is $V_{slice} = \pi(2 - \frac{2h}{h})^2\, \Delta h$.



3. Suppose that the cornerstone for a grand edifice celebrating the joy of calculus is cut in the shape shown to the right. As shown, the left side of the cornerstone is described by the equation $x = 1.333(y - 8.75)^2 - 114.083$ (which is the same as $x = 1.333y^2 - 23.3275y - 12.0252$), and the right side by $y = 18 - \frac{3}{4}x$. Find the volume of the cornerstone. (4 points)



Solution: Slicing horizontally at a height y, as shown in the figure, the volume of a slice will be $V_{slice} = (24)(w)\Delta y$. We note that the right side of the stone is given by $x = 24 - \frac{4}{3}y$. Thus, the width of a slice is $w = (24 - \frac{4}{3}y) - (1.333y^2 - 23.3275y - 12.0252)$, and the volume of the slice is $V_{slice} = 24(24 - 1.333y - (1.333y^2 - 23.3275y - 12.0252))\Delta y = 24(-1.333y^2 + 21.9945y + 36.0252)\Delta y$. Integrating to sum all slices in the range of y-values $0 \le y \le 18$, we have

$$\text{Volume} = \int_0^{18} 24(-1.333y^2 + 21.9945y + 36.0252)\, dy = 38,885\ \text{in}^3.$$