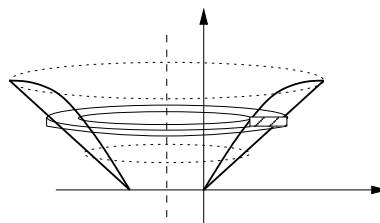


1. Consider the region bounded by $y = \sin(x)$ and $y = \frac{2}{\pi}x$ (so that $0 \leq x \leq \frac{\pi}{2}$ and $0 \leq y \leq 1$). Write (**do not evaluate**) an integral that gives the volume obtained by rotated the region around the line $x = -1$. (3 points)

Solution: The region is shown to the right. We slice horizontally, as shown. The height of each slice is therefore Δy , and each is a washer (a disk with a cutout from the middle). The outer radius is $x = 1 + \frac{\pi}{2}y$ and the inner radius is $x = 1 + \arcsin(y)$, so the volume of the slice is $V_s = (\pi(1 + \frac{\pi}{2}y)^2 - \pi(1 + \arcsin(y))^2)\Delta y$. For this region, $0 \leq y \leq 1$, so the volume is given by the integral $\int_0^1 \pi((1 + \frac{\pi}{2}y)^2 - (1 + \arcsin(y))^2) dy$.

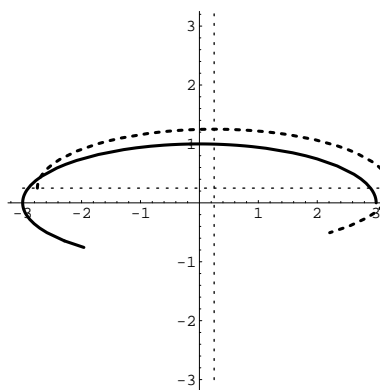


2. The integral

$$\int_0^2 \sqrt{36 \sin^2(2t) + 4 \cos^2(2t)} dt$$

gives the arclength of some curve. Sketch a curve that could lead to this integral (yes, your curve should be determined by the given integral). (3 points)

Solution: We know that arclength is given by $\int \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$, so $x'(t) = \pm 6 \sin(2t)$ and $y'(t) = \pm 2 \cos(2t)$. Antidifferentiating, we have $x(t) = \pm 3 \cos(2t) + x_0$ and $y(t) = \pm \sin(2t) + y_0$. Taking $x_0 = y_0 = 0$ and choosing the positive sign from the plus/minus, we have $x(t) = 3 \cos(2t)$ and $y(t) = \sin(2t)$, for $0 \leq t \leq 2$. This is shown in the figure to the right. If we took $x(t) = -3 \cos(2t)$ and $y(t) = \sin(2t)$ we would get the curve given by dashed line on the dotted axes offset from the origin.



3. A calculus-loving polar bear has located a triangular ice floe that appears to be perfect for ice fishing, shown to the right (lengths are in meters). To confirm this, it wants to calculate the area of the the ice floe; being a polar bear, it insists on doing it using polar coordinates; and being a calculus-loving bear, it insists on doing it with calculus. Set up an integral in polar coordinates that will give the area. (4 points)

Solution: The equation of the line in cartesian coordinates is $y = 2 - \frac{2}{3}x$. We know that $x = r \cos \theta$ and $y = r \sin \theta$, so this is the same as $r \sin \theta = 2 - \frac{2}{3}r \cos \theta$, or $3r \sin \theta = 6 - 2r \cos \theta$. Solving for r , we have $r(3 \sin \theta + 2 \cos \theta) = 6$, so $r = 6(3 \sin \theta + 2 \cos \theta)^{-1}$. Then we know that $0 \leq \theta \leq \frac{\pi}{2}$, and that the area of a polar slice of the area is $A_s = \frac{1}{2} r^2 \Delta \theta$. Thus $A = \int_0^{\pi/2} 18(3 \sin \theta + 2 \cos \theta)^{-2} d\theta$ (which, integrating numerically or calculating the area of the triangle, = 3 m²).

