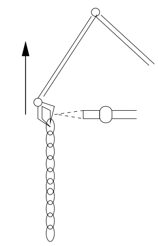
MATH 116-028 QUIZ 7 / 11 Mar 2008 Name:______________________________

1. Consider a plastic chain lifted by a mechanical arm, as suggested in the figure to the right. The chain weighs 2 g/cm and is 10 cm long. As it is lifted, a paint sprayer sprays a thick coat of fast-drying paint on it. The paint adds 0.1 g/cm to the weight of the chain. How much work is done by the mechanical arm to lift the chain 10 cm, thereby coating the entire chain? (Use calculus to find the total work. Note that in cgs units, the acceleration due to gravity, $g = 981$ cm/s^2 and force is measured in dynes.) (4 points)



Solution: Note that as the chain is lifted its mass changes. So we "slice" the work up over little distances Δy and find the work to lift the chain each such distance, then add them up using an integral. Suppose that the chain has been lifted y cm. Then y cm of the chain have been painted, and have a mass 2.1 g/cm, and $10 - y$ cm still weigh 2 g/cm. The total mass of the chain is thus $M = 2.1y + 2(10 - y)$ g. Converting to weight, we have Weight$= 981(0.1y + 20)$ dynes. Thus the work to lift the chain a distance Δy is $\Delta W = (981(0.1y + 20))(\Delta y)$. The total work is

$$\int_0^{10} 981(0.1y + 20)\, dy = 201105 \quad \text{dyne cm.}$$

2. Consider the function

$$g(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{3}, & 0 \le x < 1 \\ 1 - \frac{1}{3}x, & 1 \le x \le 3 \\ 0, & x > 3 \end{cases}$$

Is this a pdf or cdf? Why? What is the average value of x for the population this describes? (3 points)

Solution: This is a PDF, because it decreases from $x = 1$ to $x = 3$, which a CDF is not allowed to do. The mean value of the distribution is $\int_0^3 x\, g(x)\, dx = \int_0^1 \frac{1}{3}x\, dx + \int_1^3 (x - \frac{1}{3}x^2)\, dx = \frac{23}{18} \approx 1.278$.

3. Suppose that the college GPA, x, earned by squirrels living on the University of Michigan campus (which further their education by periodically enrolling in classes—you may have seen them, especially in humanities courses) is described by the density function $p(x)$. Let $P(t)$ be the cumulative distribution function corresponding to this. (3 points)

a. If $p(2) = 0.05$, what percentage of squirrels have GPAs between 1.9 and 2.1?
b. If in addition $P(2) = 0.75$, estimate $P(1.9)$.
c. If $p(x)$ is a normal distribution with mean 2.3, is $p(2)$ greater or less than $p(2.3)$?

Solution: **(a)** If $p(2) = 0.05$, then the area under $p(x)$ between 1.9 and 2.1 is approximately $(0.05)(0.2) = 0.01$, so approximately 1% of squirrels have GPAs between 1.9 and 2.1.
(b) If $P(2) = 0.75$ and we know $p(2) = P'(2) = 0.05$, then $P(1.9) \approx 0.75 - (0.1)(0.05) = 0.745$.
(c) If $p(x)$ is normal, then $p(2.3)$ is the maximum of $p(x)$, so $p(2) < p(2.3)$.