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## Some Remaining Review Problems

*(This is neither comprehensive nor guaranteed to be useful.)*

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1. Find the center of mass  $\bar{x}$ ,  $\bar{y}$  of the solid formed by the region bounded by  $x = 0$ ,  $x = 2$ ,  $y = 1 + e^{-x}$  and  $y = 1 - e^{-x}$ ,
  - a. Rotated about the  $y$ -axis, if its density is  $\delta(y) = y$  (mass units/unit volume).
  - b. Rotated about the  $x$ -axis, if its density is  $\delta(x) = x$  (mass units/unit volume).
2. If, for the region in (1a), we had  $\delta(x) = 3 + x$ , can we find  $\bar{x}$ ?  $\bar{y}$ ? Why or why not?
3. Let  $p(x)$  be a pdf, with  $a \leq x \leq b$ . Let  $Q(x)$  be an antiderivative of  $p(x)$ . Show that the cdf of  $p(x)$  is given by  $P(x) = Q(x) - Q(b) + 1$ .
4. Let  $p(x)$  be a pdf for the distribution of GPAs,  $x$ , earned by University of Michigan squirrels.
  - a. What is the domain of  $p(x)$ ?
  - b. Sketch a reasonable graph that could be  $p(x)$ .
  - c. What is the meaning of the statement  $p(2) = 0.05$ ?
  - d. If  $P(x)$  is the cdf for this distribution, what is the meaning of the statement  $P(2) = 0.953$ ?
5. Carefully determine if each of the following series converges or diverges.
  - a.  $\sum \frac{x^n}{5^n + n^2}$ , if  $|x| \leq 4$ .
  - b.  $\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln(n)}$
  - c.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \cdot \ln(n)}$
6. Carefully explain why, if  $\sum |a_n|$  converges, we are able to conclude that  $\sum a_n$  also converges.
  - a. Carefully explain why, if  $\sum a_n$  converges, we are unable to conclude that  $\sum |a_n|$  also converges.