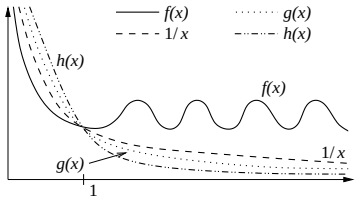
Some Random Review Problems

1. Let $F(x) = \int_2^x \frac{t^2-3}{t-1}\,dt$. For what value of a is the average value of $F(x)$ equal to $4 - \ln(3)$?

2. Consider the functions shown graphed to the right. What can you say about the convergence of



- **a.** each of $\int_0^1 f(x)\,dx$, $\int_0^1 g(x)\,dx$, and $\int_0^1 h(x)\,dx$?
- **b.** each of $\int_0^1 x\,f(x)\,dx$, $\int_0^1 x\,g(x)\,dx$, and $\int_0^1 x\,h(x)\,dx$?
- **c.** each of $\int_1^\infty f(x)\,dx$, $\int_1^\infty g(x)\,dx$, and $\int_1^\infty h(x)\,dx$?
- **d.** each of $\int_1^\infty f(x)/x\,dx$, $\int_1^\infty g(x)/x\,dx$, and $\int_1^\infty h(x)/x\,dx$?

3. Let R be bounded by $y = \sin(x)$, $y = 1/\sqrt{2}$, $x = \pi/4$ and $x = 3\pi/4$. Find the volume if R is

- **a.** rotated around the x-axis.
- **b.** rotated around $y = -2$.
- **c.** the base of a solid whose cross-sections perpendicular to the x-axis are semicircles.
- **d.** the base of a solid whose cross-sections perpendicular to the x-axis are equilateral triangles.
- **e.** challenge problem: rotated around the y-axis.

4. An airplane propeller has the shape given by $r(\theta) = 3\sin(3\theta)$ (in meters).

- **a.** What is the domain for this (range of θ values)?
- **b.** What is the area of one blade? Sketch a "slice" used to find this area.
- **c.** If the density of a blade (with $0 \le \theta \le \pi/3$) is $\delta(\theta) = 1 - \cos(6\theta)$ kg/m^3, find the mass of the propeller blade.

5. Find each of (a) $\int \frac{1}{x(x-2a)(x+b)}\,dx$; (b) $\int \frac{3}{x^2-2x+2}\,dx$; and (c) $\int (x^2+x)e^{2x}\,\sin(x\,e^x)\,dx$.

6. A pond filled with muddy water is given by the region bounded by $y = 0$, $x = 100 + \sqrt{5y}$, $y = 5$ and $x = 0$, rotated about the y-axis. Suppose the density of the muddy water is $\delta(y) = (6-y)\cdot 1000$ kg/m^3.

- **a.** Find the mass of a vertical column of the water, $1 \times 1 \times 5$ m^3 in volume.
- **b.** Therefore, deduce the pressure at the bottom of the pond.
- **c.** Find the total force exerted by the water on the bottom of the pond.
- **d.** Find the y-center of mass of all of the water in the pond.