Integration Review Problems—Solutions

For each of the following, pick the correct integration method to use in the first step of finding the antiderivative.

t. $\int \frac{x}{x^2-3x+2}\,dx$:	IBP	long division	$\boxed{\text{partial frac}}$	subst	table	trig ident
h. $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$:	IBP	long division	partial frac	$\boxed{\text{subst}}$	table	trig ident
i. $\int \frac{\sin(x)\,e^{1/\cos(x)}}{\cos^2(x)}\,dx$:	IBP	long division	partial frac	$\boxed{\text{subst}}$	table	trig ident
s. $\int \frac{dx}{\sqrt{(x+1)^2+1}}$:	IBP	long division	partial frac	$\boxed{\text{subst}}$	table	trig ident
ι**.** $\int \frac{dx}{x^{1/5}\sqrt{1+x^{4/5}}}$:	IBP	long division	partial frac	$\boxed{\text{subst}}$	table	trig ident
σ**.** $\int \cos^3 x\,\sin^5 x\,dx$:	IBP	long division	partial frac	subst	table	$\boxed{\text{trig ident}}$
u. $\int \phi\sin^2(2\phi)\,d\phi$:	IBP	long division	partial frac	subst	table	$\boxed{\text{trig ident}}$
n. $\int 1+\cos^2\theta\,d\theta$:	IBP	long division	partial frac	subst	table	$\boxed{\text{trig ident}}$
b. $\int \frac{y^4}{y^2+1}\,dy$:	IBP	$\boxed{\text{long division}}$	partial frac	subst	table	trig ident
o. $\int \ln(x^2+x)\,dx$:	$\boxed{\text{IBP}}$	long division	partial frac	subst	table	trig ident
υ**.** $\int t\,e^{t^2+1}\,dt$:	IBP	long division	partial frac	$\boxed{\text{subst}}$	table	trig ident
ν**.** $\int t\,e^{t+1}\,dt$:	$\boxed{\text{IBP}}$	long division	partial frac	subst	table	trig ident
d. $\int (t^2+1)\,e^t\,dt$:	$\boxed{\text{IBP}}$	long division	partial frac	subst	table	trig ident

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Find each of the integrals above. **t.** $2\ln|x-2|-\ln|x-1|+C$; **h.** $2\sqrt{1+\sin x}+C$; **i.** $e^{1/\cos(x)}+C$; **s.** $\ln|x+\sqrt{(x+1)^2+1}|+C$ (subst; then table); ι**.** $\frac{5}{2}\sqrt{1+x^{4/5}}+C$; σ**.** $\frac{1}{6}\sin^6 x-\frac{1}{8}\sin^8 x+C$; **u.** $\frac{x^2}{4}-\frac{1}{8}x\sin(4x)-\frac{1}{32}\cos(4x)+C$ (trig; then IBP); **n.** $\frac{3x}{2}+\frac{1}{4}\sin(2x)+C$; **b.** $\frac{y^3}{3}-y+\arctan(y)+C$; **o.** $x\ln(x^2+x)-2x+\ln|x+1|+C$; υ**.** $\frac{1}{2}e^{t^2+1}+C$; ν**.** $t\,e^{t+1}-e^{t+1}+C$; **d.** $(t^2-2t+3)\,e^t+C$

Carefully determine whether each of the following converges or not. If it converges, find its value. If it does not, explain why.

e. $\int_1^\infty \frac{x}{\sqrt{x^3+1}}\,dx$ Diverges. Compare with $\frac{1}{\sqrt{2x}}$ (or a smaller multiple).

δ**.** $\int_1^\infty \frac{\ln x}{x^2}\,dx$ Converges. $\int_1^b \frac{\ln x}{x^2}\,dx=-\frac{1}{b}\ln b-\frac{1}{b}+1\to 1$ as $b\to\infty$.

f. $\int_\pi^\infty \frac{2+\cos\theta}{\theta}\,d\theta$ Diverges. Compare with $\frac{1}{\theta}$.

μ**.** $\int_1^2 \frac{1}{1-x^2}\,dx$ Diverges. $\int_a^2 \frac{1}{1-x^2}\,dx=\frac{1}{2}\ln(3)+\frac{1}{2}\ln|\frac{a-1}{a+1}|\to-\infty$ as $a\to 1^+$.

η**.** $\int_2^\infty \frac{1}{e^{2x}-1}\,dx$ Converges. $\int_2^b \frac{1}{e^{2x}-1}\,dx=\ln(\frac{\sqrt{b^2-1}}{b})-\ln(\frac{\sqrt{e^2-1}}{e^2})\to\ln(\frac{e^2}{\sqrt{e^2-1}})$ as $b\to\infty$.

!. $\int_2^\infty \frac{1}{t\,\ln(t)^p}\,dt$, $p>0$ (hint: consider three cases.) Converges if $p>1$ (work out three cases: $p<1$, $p=1$, and $p>1$).

Please note! The problems on this review sheet are (probably) not like (or at least not exactly like) those that will show up on the midterm. However, the skills that you need to do these are (probably) those you'll need to do well on the midterm.