Math 216–040 Final, 23 April 2002 Name: _____ (2 points)

Prof. Gavin LaRose **page 1 of 4** Lab section: ____ (10,11am,2,3pm = 41-44)

Please do *not* fill in:	problem:	Name	1	2	3	4	5	6
	total pts:	2	24	14	10	22	22	6
	score:							

For all problems, *SHOW ALL OF YOUR WORK*. While partial credit will be given, partial solutions that could be obtained directly from a calculator or a guess are worth no points. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. *You do not need but may use the normal graphing calculator functions of any graphing calculator, but NOT any differential equations functionality it may have.* If you need to borrow a graphing calculator, ask me.

Note the existence of a "formula sheet" at the end of the exam.

1. Solve each of the following

   a. $xy' = 3y - 2x$, $y(1) = 2$. (6 points)

   b. $yy' = xy^2 + 2x$, $y(0) = 1$. (6 points)

   c. $y''' + 4y' = 2x + 3e^{-x}$. (6 points)

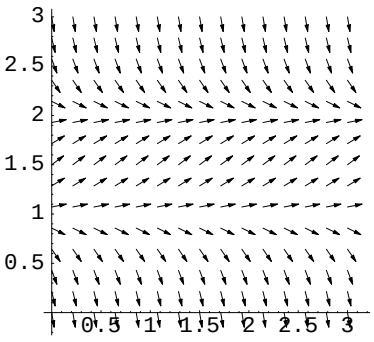
   d. $x' = x - 4y$, $y' = -2x + 3y$. (6 points)

2. Find or solve each of the following.
   a. *Use the definition* of $\mathcal{L}\{f(t)\}$ to find $\mathcal{L}\{f(t)\}$, where $f(t) = 3(u(t-2) - u(t-4))$. (4 points)

   b. Find $\mathcal{L}^{-1}\{\frac{1}{(s+2)(s^2+4s+13)}\}$ (4 points)

   c. Solve with Laplace transforms: $y'' + 9y = \begin{cases} 2, & t < 4 \\ 0, & t \geq 4 \end{cases}$, $y(0) = y'(0) = 0$. (6 points)

3. Suppose that the direction field to the right corresponds to a differential equation $\frac{dy}{dx} = f(y)$.
   a. Draw a phase diagram for the differential equation. What does it tell you about solutions to the differential equation? (5 points)



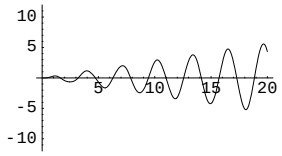
   b. Write a differential equation that could generate this direction field. (5 points)

4. An electrical circuit with inductance L, resistance R and capacitance C is modeled by the differential equation $LQ'' + RQ' + \frac{1}{C}Q = f(t)$, where $Q(t)$ is the charge on the capacitor in the circuit and $f(t)$ is the applied voltage. The units of inductance are henries (h), resistance is measured in ohms (Ω), and capacitance in Farads (F).

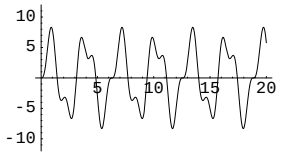
   a. If $R = 0$, $L = 0.5$h and $C = 0.08F$, find Q_c. (4 points)

   b. Match each of the responses (solutions to the differential equation) with the correct applied voltage. For each explain in one sentence why it corresponds to the response you indicate. (8 points)

Response 1

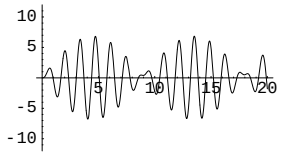


Response 2

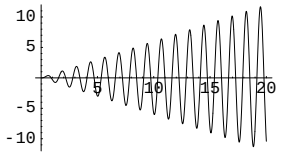


Applied voltages:
A. $f(t) = 64\sin(2t)$
B. $f(t) = 12\sin(4.3t)$
C. $f(t) = 3\sin(5t)$
D. $f(t) = 3t\sin(2t)$

Response 3



Response 4



   c. Now take $L = 0.5$h, $R = 4\Omega$, and $C = 0.08F$. Suppose that initially there is no charge on the capacitor but that there is a current of 2 amps in the circuit (that is, $I(0) = \frac{dQ}{dt}\big|_{t=0} = 2$). If the system is struck by lightning at $t = 5$ seconds, introducing a 500V spike to the circuit, write an initial value problem modeling the system. (4 points)

   d. Solve your problem from (4c). (6 points)

5. The Lotka-Volterra equations modeling a pair of interacting populations is

$$\begin{aligned} x' &= -ax + bxy \\ y' &= \quad gy - dxy \end{aligned}$$

(where $a, b, c, d > 0$).

a. What type of interaction between the two populations is being modeled? How do you know? (4 points)

b. If $a = 1$, $b = 1$, $d = 1$ and $g = 4$, find and graph the nullclines for the system. What information about the populations do they tell you? (6 points)

c. If $a = 1$, $b = 1$, $d = 1$ and $g = 4$, what do you expect to happen to the populations? Explain. *(You must do some calculations to correctly answer this question.)* (12 points)

6. The Infamous Tortoise of Questionable Assertions has returned for the final. Its assertion for the final is the following: "For some linear, nonhomogeneous, second-order ordinary differential equation, a general solution could be given by $y = c_1 t^\alpha + c_2 t^\alpha + y_p$, where c_1, c_2, and α are distinct constants and y_p is a particular solution to the ODE." What parts, if any, of the Tortoise's assertion are correct? What parts, if any, are incorrect? (6 points)