

For all problems, *SHOW ALL OF YOUR WORK*. While partial credit will be given, partial solutions that could be obtained directly from a calculator or a guess are worth no points. Continue your work on the back of the page or extra sheet at the end of the exam if you need additional space. *You do not need but may use the normal graphing calculator functions of any graphing calculator, but NOT any differential equations functionality it may have.* If you need to borrow a graphing calculator, ask me.

1. Solve each of the following to find, where possible, explicit real-valued solutions.

a. $y' + 2xy = 3x^2y$, $y(1) = 5$. (6 points)

b. $y''' + 4y'' + 5y' = 0$. (5 points)

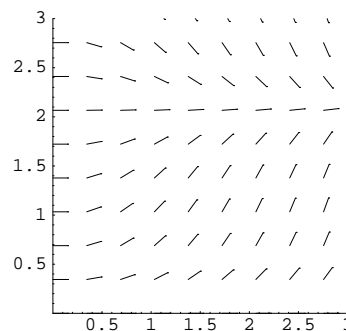
c. $y'' + 6y' + 9y = 0$, $y(0) = 0$, $y'(0) = 4$. (6 points)

d. $y' = \tan(x)y - 3$. (5 points)

2. The figure to the right shows the direction field for some equation

$$\frac{dy}{dx} = f(x, y)$$

Use this and *Euler's Method* with a step size of $h = 1.0$ to estimate $y(3)$ if $y(0) = 1$. Be sure that it is clear from your work how you obtain your answer, and how it is related to the numerical method you are using. (10 points)



3. Consider the differential equation $y'' = -2yy'$, $x \neq -1$.
- Verify that $y_1 = \frac{1}{2}$ and $y_2 = \frac{1}{x+1}$ are both solutions to this. (6 points)

b. Are these two functions linearly independent? Why or why not? (5 points)

c. Can you write a general solution to this problem? Explain. (4 points)

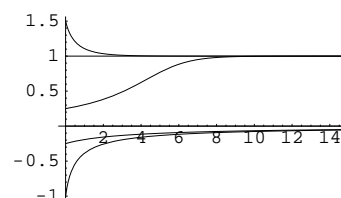
4. A not-so-cool-as-all-that cat lounges languidly in a room $9 \times 10 \times 5$ meters large. The air flow in the room introduces clean air at a rate of $1\text{m}^3/\text{min}$, and the cat's smoky cigarette converts 0.002m^3 of air per minute into a mixture containing 5% carbon monoxide. This is well mixed in the room, and the ventilation system removes 1m^3 of smoky air from the room every minute. Find the amount of carbon monoxide in the room as a function of time. (16 points)

5. Below are three phase diagrams for differential equations of the form $\frac{dP}{dt} = f(P)$.



- a. Which, if any, of these phase diagrams could correspond to the differential equation $\frac{dP}{dt} = P(1-P^2)$? Why? (6 points)

- b. Which, if any, of these phase diagrams could correspond to the plot of solution curves to a problem of this type shown to the right? (6 points)



6. Rewrite each of the following as indicated. Be sure to show all of your work. Yes, every little step.

a. $z = \frac{2i}{(1-3i)}$ as $z = x + iy$. (4 points)

b. $z = -4 + i$ as $z = re^{i\theta}$. (4 points)

7. Archimedes, brilliant man that he was, told us that the buoyancy force on an object is equal to the weight of the water it displaces. For a cylindrical buoy suspended with its axis perpendicular to the surface of the water, this means that $F_B \propto y$, where y is the amount the buoy is submerged beyond its equilibrium position. Suppose that we have a buoy with mass m for which $F_B = 576y$ (in English units).

a. Explain why the equation $y'' + \frac{576}{m}y = 0$ is a good model for this problem. (5 points)

b. Suppose that a playful mermaid pulls the buoy down slightly to start it oscillating. What is the mass of the buoy to be if its period of oscillation is π seconds?¹ (12 points)

¹ : could the mermaid beat you in an arm-wrestling contest? (No, you don't have to answer this question to get full credit on the problem. . .)