LAB 2: FIRST ORDER SYSTEMS AND THE VAN DER POL OSCILLATOR, PART B

1. MATLAB

MATLAB commands we use in this lab include the following.

1.1. **dirfieldsys2.** Plots the direction field for a system
>> dirfieldsys2(f_handle, g_handle, [xmin, xmax], [ymin, ymax])
The function handles will be interpreted with all vector operations replaced by
the element-by-element operators (.*, ./ and .^).

1.2. **ode45.** Finds a numerical approximation to a differential equation or system of equations; for a system
>> [tsol,xsol] = ode45(@(t,x) [$rhs1$;$rhs2$], [tmin,tmax], [x0;y0]);

1.3. **ode15s.** Finds a numerical approximation to a differential equation or system of equations; appropriate when the system is stiff (that is, has a variable that changes very rapidly. For a system,
>> [tsol,xsol] = ode15s(@(t,x) [$rhs1$;$rhs2$], [tmin,tmax], [x0;y0]);

1.4. **plot.** Plot one vector against another. If we're plotting a phase portrait given output from ode45,
>> plot(xsol(:,1), xsol(:,2));

2. BACKGROUND

In this lab we are considering the van der Pol oscillator, a model of an active RLC circuit with a nonlinear resistor that dissipates energy when the amplitude of the current is high, and pumps energy into the system whenever the amplitude of the current is too low. The resulting differential equation is

$$(1) \qquad x'' + \mu(x^2 - 1)x' + x = 0,$$

which we are calling "the" van der Pol equation. In Exercise 2 in the prelab you wrote this as a system of equations, and at the end of the prelab we linearized the system to obtain the linearized version of the equation,

$$(2) \qquad \begin{aligned} x' &= y \\ y' &= -x + \mu y. \end{aligned}$$

Your and your partner are responsible for completing the Reflection described in section 5. The goal is for you to be able to complete that by the end of this lab period; if you do not fully complete it by then, be sure to check with your GSI to determine when you will submit it.

3. Exercise 1

Review your work on Part A, Exercise 2 of the lab. Make sure that you understand how the dynamics of the (nonlinear) system are modeled by its linearization, and what the linearization is and is not able to tell you about the behavior of the nonlinear system.

Find the eigenvalues and eigenvectors for the coefficient matrix of the linearized system when $\mu = 0$, $\mu = 0.1$, and $\mu = 1$. How will the phase portraits for these be different (sketch them by hand.)? Then $\mu = 0$ and generate a phase portrait for the nonlinear system. Notice how this reflects the behavior you see in the linearized system, and why the linear and nonlinear systems are in agreement in this case.

4. Exercise 2

Thinking about what the phase portrait for $\mu = 1$ looks like (Part A, Exercise 2) and that for $\mu = 0$ (Exercise 1, above), what do you expect the phase portrait to look like for $\mu = 0.1$? Generate a phase portrait by plotting trajectories with small and larger initial conditions to determine how your intuition is correct (or perhaps slightly off).

5. Reflection

Your reflection should be completed by you and your partner collaboratively, and should be about a page in length, including 2–3 figures to illustrate your conclusions. To generate this, first write down (or find) the nonlinear van der Pol system and its linearization. Then follow the following steps:

(1) Take two minutes to think, on your own, about the answers to the following questions:
   (a) How is the linearized system obtained from the nonlinear system? What does this tell you about what it should be able to tell you about the behavior of the nonlinear system?
   (b) How is the description of the nonlinear resister in the Prelab and background sections of Parts A and B reflected in the phase portraits that you generated in this lab?
   (c) In the van der Pol equation, x is a current in the circuit. What will a graph of x look like as a function of time, t, given your phase portraits for the linear and nonlinear systems?
(2) Discuss these three questions with your partner, and, if you wish, the others at your table, by having each person of your group give their answers and reasoning. Once everyone has commented, come to a consensus as a group as to what the answers should be.
(3) With your partner, generate a reflection writeup that concisely answers the questions above.

References

[BB] James Brannan and William Boyce, *Differential Equations: An Introduction to Modern Methods and Applications*, 2015 (3rd edition), Wiley.