Math 216–S19 Written Homework 1

Instructions: Solve each of these problems. Your solution should be complete and written out in complete sentences. Where graphs are needed, you may include a print-out of output from *Matlab* (or another program, if you prefer).

1. In lab, we consider the Gompertz equation, $y' = r\,y\,\ln(K/y)$ and simplifications of that using the Taylor expansion for $\ln(y)$ near $y = K$. Here we consider the Gompertz equation as well as the order $n = 0$ through $n = 3$ approximations to the equation found in Exercise 4 of the prelab.

   (a) Find the critical points of all four equations and determine their stability.

   (b) Solve the Gompertz equation, and the linear and quadratic approximations (that is, the approximations of order $n = 1$ and $n = 2$) exactly.

   (c) Recall that we expect the approximate equations to be valid when y is near the expansion point. Find the solution to each of the equations you solved in (b) with the initial condition $y(0) = 0.8K$. Plot the solutions on the same graph and determine how they are similar and different.

2. Problem 11 in §2.4 of Brannan and Boyce (p.79 in the 3rd ed. text). Also complete parts (a)–(c), below.

   (a) Solve the equation with the initial condition $y(0) = 1$ (you will be able to find an implicit equation for y).

   (b) Based on your answer to problem 11, find the range of t and y values on which you would expect the solution to exist. (Consider t values both greater and less than 0; to find exact values you will need to solve the equation you obtain numerically.)

   (c) Use *Matlab* or some other tool to draw the direction field for the equation on an appropriate domain. Sketch the solution you found in (a) on the direction field.

3. Problem 5 in §2.5 of Brannan and Boyce (p.91 in the 3rd ed. text).

4. Problem 29 in §3.1 of Brannan and Boyce (p.129 in the 3rd ed. text). Also complete parts (a)–(c) below.

   (a) If we are solving $\mathbf{A}\mathbf{x} = \mathbf{0}$, how many solutions for $\mathbf{x}$ will there be? What are they? Explain how this is related to the eigenvalue calculation you did.

(b) Suppose the matrix is $\mathbf{A} = \begin{pmatrix} 1 & -3 \\ -5 & -1 \end{pmatrix}$. Find the eigenvalues and eigenvectors in this case. How many solutions are there to $\mathbf{A}\mathbf{x} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$? Explain how this is related to the eigenvalue calculation.

(c) Suppose the matrix is $\mathbf{A} = \begin{pmatrix} 1 & a \\ -5 & -1 \end{pmatrix}$. For what a, if any, is there a repeated eigenvalue? What are the eigenvalues if a is less than the value(s) you found? If a is greater than the value(s)?