Math 216–S19 Written Homework 3

Instructions: Solve each of these problems. Your solution should be complete and written out in complete sentences. Where graphs are needed, you may include a print-out of output from *Matlab* (or another program, if you prefer).

1. In lab 3 we consider the nonlinear system

$$N' = \gamma(A - N(1+P)), \quad P' = P(N-1).$$

We continue our analysis of this system here.

   (a) Find all critical points of this system.

   (b) Find a linear system that approximates the nonlinear system at each of the critical points you found in (a). To do this, let $(N, P) = (N_0, P_0) + (u, v)$ (where (N_0, P_0) is a critical point, and $|u, v| \ll 1$. Plug into the equation and discard nonlinear terms (which may be considered to be negligible).

   (c) Determine the type and stability of the critical point $(1, A-1)$ in the cases $0 < A < 1$ and $A > 1$. You may assume that $\gamma < \frac{4(A-1)}{A^2}$ when $A > 1$.

   (d) Sketch a phase portrait for the linearization at $(1, A-1)$ for the each of the cases $0 < A < 1$ and $A > 1$.

2. Problem 16 in §4.2 of Brannan and Boyce (p.227 in the 3rd ed. text).

3. Consider the problem given in Problem 17 in §4.6 of Brannan and Boyce (p.273 in the 3rd ed. text). Complete parts (a)–(b) and parts (c)–(e) below

   (c) Explain how what you are seeing is related to the phenomenon of beats.

   (d) Write the solution for $y(t)$ in the form $R\cos(\omega t) + C\cos(\omega_0 t - \delta)$.

   (e) Plot the amplitude of the response to the forcing, $|R|$, as a function of ω.

4. Problem 19 in §4.7 of Brannan and Boyce (p.281 in the 3rd ed. text). Write out the expression you use for y_p, and the equations you have to solve to find u_1' and u_2'. Solve for u_1' and u_2', then integrate to find the solution.