Math 216–W19 Written Homework 5

Instructions: Solve each of these problems. Your solution should be complete and written out in complete sentences. Where graphs are needed, you may include a print-out of output from *Matlab* (or another program, if you prefer).

1. Problem 9 in §6.4 of Brannan and Boyce (p.419 in the 3rd ed. text). Complete part (a) *(Hint: find eigenvalues and eigenvectors of the coefficient matrix with Matlab or some other tool.)* and then (b) and (c) below.

   (a) Are there any initial conditions for which the solution to the system will become unbounded? Explain.

   (b) Are there any initial conditions for which all solutions will decay to zero? Explain.

2. Problem 27 in §7.2 of Brannan and Boyce (p.476 in the 3rd ed. text). *(It may be useful to restate the hint as "to show almost linearity, notice the work on p.469.")*

3. In lab 5 we consider the Lorenz equations

$$\begin{aligned} x' &= \sigma(-x+y) \\ y' &= r\,x - y - xz \\ z' &= -b\,z + xy \end{aligned}$$

   In the following, take $\sigma = 10$ and $b = \frac{8}{3}$.

   (a) Find all of the critical points for the Lorenz system in terms of the parameter r. For what values of r is there only one critical point? More than one?

   (b) Find the Jacobian for the Lorenz system.

   (c) Use your work from (b) to find a linear system approximating the Lorenz system at the critical point $(0, 0, 0)$. Find the eigenvalues of the coefficient matrix of the system and determine the values of r for which the critical point is stable and the values for which it is unstable (consider $r > 0$ only).

4. Problem 10 in §7.3 of Brannan and Boyce (p.487 in the 3rd ed. text). *(Note in part (c), the problem should say "consider the trajectory that leaves the critical point $(3, 0)$." For (c), first think about what the trajectory will look like. Then try solving the problem numerically with different values of γ to see what values give trajectories on either side of the trajectory you want and use those to estimate the γ that will work. You may not be able to get an exact value.)*