Let R be a Noetherian ring and let $(*)$ $0 \to A \to B \to C \to 0$ be a short exact sequence of modules of finite length over R. We show that if $B \cong A \oplus C$ then $(*)$ splits.

Since $B \cong A \oplus C$, we have that $\mathrm{Hom}_R(C, B) \cong \mathrm{Hom}_R(C, A) \oplus \mathrm{Hom}_R(C, C)$. Hence, $\ell\big(\mathrm{Hom}_R(C, B)\big) = \ell\big(\mathrm{Hom}_R(C, A)\big) + \ell\big(\mathrm{Hom}_R(C, C)\big)$, where ℓ denotes length. Write $f : B \to C$ for the map in $(*)$. Apply $\mathrm{Hom}_R(C, \_)$ to the sequence $(*)$. From the left exactness of $\mathrm{Hom}_R(C, \_)$ we obtain that the sequence

$$0 \to \mathrm{Hom}_R(C, A) \to \mathrm{Hom}_R(C, B) \to \mathrm{Hom}_R(C, C)$$

is exact, where the map $\mathrm{Hom}(C, f) = f_*$ is given by $g \mapsto f \circ g$. Let N denote the cokernel of f_*, which evidently has finite length. It follows that

$$0 \to \mathrm{Hom}_R(C, A) \to \mathrm{Hom}_R(C, B) \to \mathrm{Hom}_R(C, C) \to N \to 0$$

is exact, and, hence, that $\ell(N) = \ell\big(\mathrm{Hom}_R(C, C)\big) - \ell\big(\mathrm{Hom}_R(C, B)\big) + \ell\big(\mathrm{Hom}_R(C, A)\big) = 0$, so that $N = 0$. Hence, $f_* : \mathrm{Hom}_R(C, B) \to \mathrm{Hom}_R(C, C)$ is surjective, which shows that the sequence $(*)$ splits. $\square$