

Math 614 Syllabus, Fall 2020

Categories and functors
Natural transformations
Equivalence and antiequivalence of categories
Products and coproducts (direct sums) in a category
Limits (inverse limits) and colimits (direct limits) in a category
Representable functors
The Zariski topology
Localization of rings and modules
Tensor products of modules and coproducts of algebras
Base change and restriction of scalars
The notion of dimension
Integral extensions,, including the lying over, going up and going down theorems
Integrally closed domains
Noether normalization over a field and over a domain
Hilbert's Nullstellensatz
Introduction to affine algebraic geometry
Antiequivalence of the category of reduced finitely generated algebras over an algebraically closed field with the category of closed algebraic sets of affine spaces and regular maps
Noetherian rings and modules
Flat, projective and free modules
Primary decomposition for ideals
Modules of finite length
Artinian rings and their structure theory
Discrete valuation rings
Characterization of normal Noetherian domains
Dedekind domains
Divisor class groups
Krull's principal ideal theorem and related dimension theory
The Artin-Rees lemma
Completion of Noetherian rings and modules, including flatness of completion