

Math 614, Fall 2020

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Prerequisites. The normal prerequisite is a one year graduate course in algebra (at Michigan, Math 593 and 594), including basic definitions about rings and modules, the structure theory of modules over a principal ideal domain, the derivation of Jordan and rational forms for matrices from this theory, the fact that a polynomial ring over a unique factorization domain is a unique factorization domain, the definition and basic properties of tensor products of modules and exterior powers of modules (this will be reviewed in the notes, but not much class time will be spent on it), and some notions from field theory (transcendence degree of a field extension, and the fact that a finite algebraic extension $K \subseteq L$ is separable if and only if the trace map $L \times L \rightarrow K$ such that $(u, v) \mapsto \text{tr}_{L/K}(uv)$ gives a nondegenerate bilinear form.) Some of this material is reviewed in the Lecture Notes.

The students need to be able to do proofs in an abstract context. We will not be using much from the group theory nor from Galois theory, except for the notions of trace and separability, and these are only needed “locally,” i.e., in a few lectures. Because we study the Zariski topology on the set of prime ideals of a ring and on the closed algebraic sets in K^n over an algebraically closed field K , some knowledge of general topology is needed, including the notions of T_0 , T_1 , and T_2 spaces (the last is the same as a Hausdorff space), compactness (open covers have finite subcovers), connectedness, and connected components. We also need the definition of continuous maps for abstract topological spaces. We will use the term “quasicompact” (instead of “compact”) for topological spaces that are not necessarily Hausdorff such that every open cover has a finite subcover. Again, this material is reviewed in the notes, but will not be covered in lecture.

I will be distributing a form prior to the start of classes asking about prerequisites.

Lecture Notes. The preliminary version of the lecture notes for this course is available as a .pdf file from the Web Page for this course

<http://www.math.lsa.umich.edu/~hochster/614F20>

Alternatively, there is a link for this course on my homepage,

<http://www.math.lsa.umich.edu/~hochster>

These notes will be revised and considerably enhanced as the lectures are given.

Attendance and remoteness issues. Attendance is not required. Synchronous lectures will be given using Zoom (I will be writing on an iPad) and these will be recorded and available through Canvas. I am happy to answer questions either by e-mail or to have one-on-one meetings (the equivalent of office hours) either by Skype or Zoom, but you will have to make an appointment, probably a day ahead of time, for a video meeting.

Grades. Grades will be based primarily on problem sets: the latter will be distributed, roughly, every other week. There will be approximately 30 problems in all, six per assignment, together with some additional problems designated “Extra Credit.” I prefer that the problem sets be submitted as .pdf files using some kind of word processor to formulate them, and this will be *required* from the second assignment on. I recommend the use of some form of T_EX (LaTeX is the most commonly used version now). Papers submitted to mathematics journals are often required to be in LaTeX. I am happy to help with questions about using T_EX or LaTeX.

I encourage you to get to know the other students in the course and to form small work groups. It is perfectly fine if you work together on some of the problems (but not on the Extra Credit problems).

Please indicate, with your solutions to problem sets, others with whom you have worked on the problem or other sources and resources you have used in formulating your solution. You may **not** use Web based resources aimed at specific problems in your solutions to problem sets. You may use material from books and from the Web such as texts and lecture notes for the purpose of helping you understand better material related to the problems.

Solutions to Extra Credit problems should be entirely your own work.

There will also be four short answer quizzes that will be given on the honor system. The questions will be True-False or “fill in the blanks.” When administering these in class I usually allow fifteen minutes, but you will have about a day and a half to submit your answers (but, despite the extra time, you may not look for the answers once you have seen the quiz questions). Quizzes will be distributed by e-mail by noon Eastern time on September 23, October 21, November 18, and December 2, all of which are Wednesdays, and due by midnight Eastern time of the following day, Thursday.

The main purpose of these quizzes is to help people to keep up-to-date, to place emphasis on certain course material, and to let me know when someone appears to be struggling with the material, perhaps because of lack of prerequisites.