Math 711, Fall 2004 **Problem Set #3**
Due: Wednesday, November 10

1. Let $R \to S$ be local homomorphism of quasilocal rings such that S is a module-finite extension of R. Prove that $S^{\mathrm{h}} \cong R^{\mathrm{h}} \otimes_R S$, or give a counterexample.

2. Show that R^{h} is Cohen-Macaulay (respectively, regular) if and only if R is Cohen-Macaulay (respectively, regular), and that R and R^{h} have the same type (the type is the dimension of the annihilator of the maximal ideal modulo some (respectively, any) system of parameters).

3. Suppose that S is flat over R, that R, S are local Noetherian rings, and that $R \to S$ is local.

(a) Show that if $R \to S$ is formally smooth, then S is reduced if and only if R is reduced. (You may assume that a formally smooth algebra over a field is reduced.)

(b) Show that by example that S need not be a domain even if S is pointed étale over R and R is a domain.

4. Let S and T be R-algebras. Is it correct that $\Omega_{S \otimes_R T/R} \cong T \otimes_R \Omega_{S/R} \oplus S \otimes_R \Omega_{T/R}$? Prove your answer.

5. Let (R, P, K) be a quasilocal ring and let M be an $s \times s$ matrix over R that is congruent to the identity matrix mod P. Prove that if n is a positive integer not divisible by the characteristic of K, then M has an nth root over a pointed étale extension of R.

EXTRA CREDIT Let (R, P, K) be a quasilocal ring and let N be the ideal of all nilpotent elements of R. Suppose that R/N is Henselian. Is R necessarily Henselian? Prove your answer or give a counterexample.