Math 711, Fall 2005 **Problem Set #1**
Due: Wednesday, October, 12

1. Let R be a finitely generated $\mathbb{Z}$-algebra, and suppose that for every prime integer $p \geq 2$, R/pR is an integral domain. Show that R need not be an integral domain.

2. In problem **#1**, suppose that R is also assumed to be torsion-free over $\mathbb{Z}$. Is it then true that R must be integral domain? Suppose instead that we assume that the rings of the form R/pR are all integral domains of the same Krull dimension. Is it true that R must be a domain in that case?

3. Prove that if S is an R-algebra and $N \to M$ is an S-linear map that is pure over S, then the map ${}_RN \to {}_RM$ obtained by restriction of scalars from S to R is pure over R.

4. Show that if $N \subseteq M$ are R-modules and M/N is R-flat, then $N \to M$ is pure.

5. Let (R, m, K) be an F-pure equidimensional local ring with a perfect residue class field, and suppose that R_P is Cohen-Macaulay for all prime ideals $P \neq m$. In particular, R is reduced. Let $R^{1/p}$ be the ring obtained by adjoining pth roots of all elements of R. Assume that $R^{1/p}$ is finitely generated as an R-module. Prove that $R^{1/p}/R$ is a maximal Cohen-Macaulay R-module. (D. Hanes.)