Math 711, Fall 2005 **Problem Set #3**
Due: Wednesday, November 16

1. Let $R \hookrightarrow S$ be a homomorphism of rings of prime characteristic $p > 0$ such that $R \to S$ splits. Show that if S is F-split, then so is R.

2. Consider an $r \times s$ matrix of indeterminates X over a field K, and give $K[X]$ the induced ASL structure from $K\big[[X \,|\, Z/r]\big]$, where $[X \,|\, Z]$ is an $r \times (r+s)$ matrix of indeterminates whose first s columns give X. Let $1 \leq i_1 < \cdots < i_t \leq r$ and $1 \leq j_1 < \cdots < j_t \leq s$ be integers, where t is an integer with $1 \leq t \leq \min\{r, s\}$. Consider the ideal J of $K[X]$ generated by all minors h of X of varying size such that h is not $\geq \delta = X[i_1, \ldots, i_t \,|\, j_1, \ldots, j_t]$. Describe J as explicitly as you can. Must $K[X]/J$ be Cohen-Macaulay? Why?

3. Let $F_1, \ldots, F_k$ be a regular sequence of forms of positive degrees $d_1, \ldots, d_k$ in the polynomial ring $K[x_1, \ldots, x_n]$. What is the $\mathfrak{a}$-invariant of $K[x_1, \ldots, x_n]/(F_1, \ldots, F_k)$?

4. Let $R = K[X, Y, Z]/(X^3+Y^3+Z^3) = K[x, y, z]$ and let $S = K[xs, ys, zs, xt, yt, zt] \subseteq R[s, t]$, where s and t are new indeterminates. Show that $R[s, t]$ is Cohen-Macaulay and S is a direct summand of $R[s, t]$ as an S-module, but S is not Cohen-Macaulay.

5. Let K be a field, and let X be an $r \times s$ matrix over K, where $r \leq s$. Let $I = I_r(X)$, the ideal generated by the maximal size minors of X. Determine the $\mathfrak{a}$-ivariant of $K[X]/I$. (Problem **2.** of Problem Set #2 may be helpful.)