Math 711, Fall 2005 **Problem Set #4**

Due: Wednesday, November 30

1. Suppose that Δ is the union of two simplicial subcomplexes Γ_1 and Γ_2 such that Γ_1 and Γ_2 have dimension n, while $\Gamma_1 \cap \Gamma_2$ has dimension $n-2$. Prove that $K[\Delta]$ is not Cohen-Macaulay for any field K.

2. Let $I_1, \ldots, I_d$ be a sequence of flat ideals of the ring R such that for all k, $1 \leq k < d$, $I_{k+1} \cap (I_1 + \cdots + I_k) = I_{k+1}(I_1 + \cdots + I_k)$. Show that the total complex of the tensor product of the d complexes $0 \to I_j \to R \to 0$ is acyclic.

3. Let R be a Noetherian local domain of mixed characteristic $p > 0$, and let I be any ideal of R that contains p. Show that $\mathrm{Rad}\,(IR^+)$ has finite Tor dimension over R^+.

4. Let R be an equicharacteristic local ring that has a nonzero module of finite length and finite projective dimension. Prove that R must be Cohen-Macaulay.

5. Let R be an equicharacterisitc Noetherian ring, and M a module of finite projective dimension over R. Let $R \to S$ be a homomorphism of Noetherian rings and let $I = \mathrm{Ann}_R M$. If $IS \neq S$, show that $\mathrm{height}\,(IS) \leq \mathrm{pd}_R M$.