Corrections to Classical Algebraic Geometry: a modern view

September 13, 2022

p. 34, l.12: two functions (g, h)

p. 97, l.-11:$V(\sum \alpha_i t_i^2)$.

p. 102, l.-5: replace PGL(4) with PGL(3).

p. 115, (3.3): add 4 as a factor in the numerator.

p. 115, l.-4: $f \mapsto 4b/27$.

p. 119, (3.11) add the factor $2^6.3^3 = 1728$.

p.126, Corollary 3.2.5: replace C with $\mathrm{He}(C)$.

p.141, l.4:

$$H(6aP + bQ) = (-2Ta^3 + 48S^2a2b + 18TSab^2 + (T^3 + 16S^2)b^3P$$
$$+(8Sa^3 + 3Ta^2b - 24S^2ab^2 - TS^2b^3)Q,$$

where the product of a covariant and a contravariant is considered as the composition of the corresponding equivariant maps.

p.141, l.8: $Q(f)$ is the locus of lines which intersect C at three points such that the polar line of the Hessian curve $H(f)$ with respect two of the points is tangent to $H(f)$ at the third point (see [384]). This is similar to the property of the Pippian which is the set of lines which intersect C at three points such that the polar line with respect to two of the points is tangent to C at the third point.

p.141: Exercise 3.2 (ii) replace C with $\mathrm{Cay}(C)$.

p.141: Exercise 3.3 (i) set $K(\ell)$ of second polars p.142, Exercise 3.8: replace $\mathbb{T}_a(\mathrm{He}(C))$ with $\mathbb{T}_b(\mathrm{He}(C))$, where b is the singular point of $P_a(C)$.

p.142, Exercises 3.9 and 3.10: replace $\mathrm{Cay}(C)$ with the dual of $\mathrm{Cay}(C)$.

p. 174, l.-3:replace k with b.

p. 176, Theorem 4.2.17: replace k with b, 2^{2g+k-1} with 2^{g+k-1} and the last line should be $2^{g+b-1}(2^g-1)$.

p. 181, l. -1: $= \prod_{0\le i\le m-r-1} \frac{\binom{m+i}{m-r-i}}{\binom{2i+1}{i}}$.

p. 211, Proposition 5.4.17: congruent to g modulo 4.

p. 215, l. 17: $T(x) = \mathbb{T}_x(C)\cap C$.

p. 217, the line above (5.38): $p_1^*(\theta) = p_2^*(\theta)$, restricted to R. In (5.38): replace the equality by the numerical equality.

p. 231, Theorem 6.1.9, equation (6),(7):replace u_i with t_i and a_{01} with a_{0i}.

p. 238, l.-16: delete "an unramified irreducible cover if".

p. 245, Proposition 6.3.3. Replace the first paragraph of the proof with the following one.

If we have fewer than eight base points, then all nonsingular quadrics share the same tangent line at a base point. This implies that N_θ contains a quadric Q with a singular point at a base point. The computation of the tangent space of the discriminant hypersurface given in (1.4.5) shows that Q is a singular point of the discriminant curve C, a contradiction.

p. 265, l.14: $(abc)^4$.

p. 267: equation (iv) should be $t_2^4 + g_4(t_0,t_1) = 0$.

p. 270: first formula after the Table: $e^{-i\pi/4}\to e^{-\pi i/3}$ and $(x-y)\to(x-iy)$.

p. 276, Exercise 6.5: The result is known and the number is equal to 12. There are two isomorphism classes of curves with this number of inflection bitangents (A. Kuribayashi, K. Komiya, Hiroshima Math. J. , 7 (1977), 743–768).

p. 299: Replace the first paragraph with the following.

It is isomorphic to the blow-up of the union of coordinate subspaces of codimension 2. The action of the torus $(\mathbb{C}^*)^{n+1}$ on $\mathbb{P}^n$ (by scaling the coordinates) extends to a biregular action on X. In the case $n=2$, the toric surface X is a del Pezzo surface of degree 6 isomorphic to the blow-up of 3 points in the plane, no three of which are collinear. In $n>2$ the variety is singular.

p. 297,l.-11:$e_0,\dots,e_n$

p. 300, l. 10: trisecant line

p. 309, l. -9: replace ϕ with T.

p. 317, l. 20-23: replace $\overline{xq}$ with $\overline{xq'}$, where q' is the residual point of the tangent line of F(x) at q.

p. 319, l.15-16: replace $m-2$ with $m-1$.

p. 326, l.-14: $t_{x,y}\mapsto e_{x,y}$.

p. 356: Proposition 8.1.23 (ii) Since $K_S^2>1$.

p. 368. l. 13: $W(\mathbf{E}_4)\cong\mathfrak{S}_5$.

p. 371, l. -13 The group $\mathfrak{S}_5$ acts on $\Lambda^2\mathbb{C}^5$ via its irreducible representation on $\mathbb{C}^5$. The character of this action is the same as follows in the text.

p. 371, l. 3 (after Table): k+1 non-intersecting (-1)-curves.

p. 392, l.15: containing S.

p. 393, last line: last row in s_1 is $(0,0,1)$.

p. 394, Replace l.13-l.-3 with Using the character table of $\mathfrak{S}_5$, we find that χ is the character of an irreducible 6-dimensional irreducible representation isomorphic to $V=\bigwedge^2 R_{\mathrm{st}}$, where R_{st} is the standard 4-dimensional irreducible linear representation of $\mathfrak{S}_5$ with character vector $(4,2,1,0,-1,0,-1)$ (see [**?**], p. 28). The linear system $|-K_S|$ embeds S in $\mathbb{P}(V)$. Since V is isomorphic to its dual representation, we can identify $\mathbb{P}(V)$ with $|V|$.

We will see later in Chapter 10 that $G_1(\mathbb{P}^4)$, embedded in $\mathbb{P}^9$, is defined by five pfaffians of principal minors of a skew-symmetric 5×5-matrix (p_{ij}), where $p_{ji}=-p_{ij}, i<j$, are the Plücker coordinates. The group $\mathfrak{S}_5$ acts on $\mathbb{P}^9$ via its natural representation on $\bigwedge^2 W$, where W is an irreducible representation of $\mathfrak{S}_5$ with character $(5,1,-1,-1,0,1,1)$. The representation $\bigwedge^2 W$ decomposes into irreducible representation $V\oplus R'_{\mathrm{st}}$, where R'_{st} is the standard 4-

p.395: (8.21) add $-2\sum t_i^3t_j^3$.

p.395: after (see [201]) The singular points are $[1,-1,-1],[-1,1,-1],[-1,-1,1],[1,1,1]$.

p. 396, Lemma 8.6.1: $\sum_{i=0}^n t_i^2 = \sum_{i=0}^n a_i t_i^2 = 0$.

p. 401,: $2A_1 : x_2 \succ x_1, x_4 \succ x_3$

p. 402, l.-1: $|2e_0 - \sum_{j=1}^5 e_j + e_i, i = 1, \ldots, 5$.

p. 403, Theorem 8.6.8: isomorphic either to a cyclic group of order 1, 2 or 4...

p. 404, l.-18: (i) $\cdots + at_3^2 - at_4^2$ p. 419, l.-1:$\sigma_1^2 = \sigma_2^2$.

p. 424: replace the reference [112] to A. Clebsch, Ueber den Zusammenhang einer Classe von Flächenabbildungen mit der Zweitheilung der Abel'schen Functionen, Math. Ann. **3** (1871), 45–75.

p.431, l.-6: 120 different conjugate pairs of triads.

p.444, the last paragraph in Section 9.2.1:center of the projection lies in the plane spanned by the image of the exceptional section and a ruling, or lies outside of this plane.

p.448, equation (9.18): $g_3 = t_0 t_1^2 + t_1 t_2^2 + t_2^3 = 0$.

p. 454: Table 9.2:

I	II	III	IV	V	VI	VII	VIII	IX	X	XI
72	50	24	34	60	64	52	66	60	58	42
XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX	XXI	
48	62	50	32	64	58	56	40	0	54	

Table 1: Number of determinantal representations

p. 456: The matrix should be

$$\begin{pmatrix} t_0 + t_3 & t_3 & t_3 \\ t_3 & t_1 + t_3 & t_3 \\ t_3 & t_3 & t_2 + t_3 \end{pmatrix},$$

p. 457, in (iii): $2\xi_0\xi_1 - \xi_2^2$.

p. 465: last syntheme in total T_5 is $(16, 24, 35)$.

p. 476, l.5:$H_{ij} : z_i + z_j = 0$.

p. 478, l.-5: $y_i - \frac{1}{3}(y_0 + y_1 + y_2 + y_3 + y_4)$.

p.479, l.7: Castelnuovo-Richardson quartic

p. 480: add one row (11111) in the determinant formula for I_{100}.

p. 487,l.-11: Here σ acts via the matrix $\mathrm{diag}[1,-1,i,-i]$.

p. 487,l.-4: If we choose the new coordinates to transform t_2t_3 to $t_2^2+t_3^2$, after rescaling, we get the equation $t_0^3+t_0(t_1^2+t_2^2+t_3^2)+\lambda t_1t_2t_3=0$.

p. 488: equation (9.70): $\ldots + t_3^2 t_1$.

p. 493:In Table 9.6 add the monomial $at_1t_2t_3$ to the equation on V. Also the number of Eckardt points in VII is 1 not 3.

pp. 493–495: replace Table 9.6 with Table 9.5.

p. 496, starting with line 8, replace the rest of the analysis of the case 3D with the following:

Suppose we have an automorphism $g \notin H$. If $gHg^{-1}=H$, then, replacing g with the product with some involution in H, we may assume that g commutes with σ. This shows that we can simultaneously diagonalize the matrices representing g and σ. It is immediately checked from the equation of the surface that this is possible only if $a=1$ and g is the transformation which switches t_0 and t_1. So, if $gHg^{-1}\neq H$, we obtain that $\mathrm{Aut}(S)$ is isomorphic to $\mathfrak{S}_3$ or $2\times\mathfrak{S}_3$. This gives types VI and VIII.

Let us assume that $H'=gHg^{-1}\neq H$. Then H' is the subgroup defined by the three Eckardt points y_i on the line $\ell'=g(\ell)$. Since each of the involutions corresponding to the points x_i commutes with at most one involution corresponding to the points y_i, we obtain that one of the lines $\overline{x_iy_j}$ contains the third Eckardt point and defines a subgroup of $\mathrm{Aut}(S)$ isomorphic to $\mathfrak{S}_3$ which one common involution with H. Replacing H' with this subgroup, we may assume that the lines ℓ and ℓ' intersect at $x_1=y_1$ and, hence span a plane Π. Each of the pairs of lines $(\overline{x_iy_2},\overline{x_iy_3}), i=2,3$, contains at most one line contained in S. Applying Proposition 9.1.7, we either get a complete quadrilateral in Π with 6 Eckardt points as its vertices and its three diagonals lying on S, or we get more than 9 Eckardt points on Π. Note that a plane section of S not containing a line on S intersects the 27 lines at 27 points, an Eckardt point is counted with multiplicity 3. This shows that an irreducible plane section of S contains ≤ 9 Eckardt points. If it contains a line with two Eckardt points on it, then the number is at most seven. This eliminates the second possibility. It follows from the structure of $W(E_6)$, that the first possibility gives that the four subgroups isomorphic to $\mathfrak{S}_3$ defined by the sides of the quadrilateraal generate a subgroup G of $\mathrm{Aut}(S)$ isomorphic to $\mathfrak{S}_4$. The list of maximal subgroups of $W(E_6)$ shows that either $G=\mathrm{Aut}(S)$, or $\mathrm{Aut}(S)\cong\mathfrak{S}_5$ and hence S is the Clebsch diagonal surface given by equation 9.79.

p. 496, case 2B: replace the second and the third sentence with the following. After a linear change of the variables t_0,t_1 the equation can be reduced to the form (9.68) considered in the following case.

p. 496, l.-5: If σ belongs to the conjugacy class 4B, then the equation must be as in (9.68). Then $\mathrm{Aut}(S)$ contains an additional cyclic group of type 3D. This leads to a surface of type V with

$\mathrm{Aut}(S) \cong \mathfrak{S}_4$.

p. 499, l.4:of orders $5, 2, 2$ (with $S \circ T$ of order 3) change $\eta^3 - \eta^2$ to $\eta^2 - \eta^3$.

p. 499, l.5: change $S : z \mapsto \eta^2 z$.

p. 499, l.-3:$= -20(8t_0^4 t_1 t_2 - \ldots$.

p. 501, l.1:replace C with A.

p. 501, l.11: replace Wiman sextics with nonsingular Valentiner sextics with automorphism group isomorphic to $\mathfrak{A}_6$.

p. 506: Wiman did not miss the group of order 8.

p. 509: Formula (10.3) $\sum_{i=1}^{m+1}$.

p. 510: Lemma 10.1.1, $\omega_G \cong \mathcal{O}_G(-n-1)$.

p. 512: in (10,5) $\{\mu_0, \ldots, \mu_{r+1}\}$ and

$$\mu_0 \geq \lambda_0 \geq \mu_1 \geq \lambda_1 \geq \ldots \geq \mu_r \geq \lambda_r \geq \mu_{r+1}$$

p. 520, Proposition 10.2.4. The intersection of the linear line complex $\mathfrak{C}_\omega$ with the special Schubert variety $\Omega(l)$ consist of lines intersecting l and the codimension 2 subspace $\imath_\omega(l)$.

p. 523, (10.18) add the coefficient 16 at z_4^4.

p. 532, l.-14: $i : |E| \to G(n, \bigwedge^2 E)$

p. 533, l.13: replace n with $n-1$.

p. 535, l.15: replace $\mathbb{P}^1$ with $\mathbb{P}^3$ and $\mathbb{P}^3$ with $\mathbb{P}^5$.

p. 535, l.17: replace $\langle \ell, \ell' \rangle \setminus \{\ell\}$ with $\ell' \setminus \ell \cap \ell'$.

p. 535, l.-15: $\mathcal{X} = \{(Q, \pi) \subset \mathcal{P} \times G_2(\mathbb{P}^5) : \pi \subset Q\}$

p. 536, l.17: $Jac(X)$ must be replaced with $Jac(C)$.

p. 546, Remark 10.3.20, l. 3:$e_i^* \wedge e_j^*$.

p. 547, (10.39) $p'_{il} \to p_{il}$.

p. 552, (10.42), $p'_{ij} \to p_{ij}$.

p. 560, l. 6:S_{a_1,a_1+a_2+1}; l.8: $n = a_1 + \cdots + a_k + k - 1$.

p. 563, (10.52): $K_{\mathbb{P}^3}$.

p. 563, l.-13: replace $z_1^2 - z_2z_3$ with $z_1^2 - z_2^2z_3$.

p. 574, l.3: $k_i(\check{f}) = k_{n-i-1}$.

p. 587: $\sum_{i=0}^3 a_it_i^2 = 0, \sum_{i=0}^3 b_it_i^2 = 0$.

p. 599,[149] replace Trans. Amer. Math. Soc. with Contemp. Math.

new:$\mathbb{G}^*$ replace with $\mathbb{G}^\vee$ also $\mathfrak{C}$.