GEOMETRY OF ALGEBRAIC VARIETIES

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The proposed survey is the third in a series of surveys on algebraic geometry [31, 88]. It is made up mainly from the material in Referativnyi Zhurnal "Matematika" during 1965-1973 and is devoted to the geometric aspects of the theory of algebraic varieties. After the fundamental papers of Grothendieck, effecting the harmonic connection of the purely geometric ideas of Italian algebraic geometry with the methods of commutative and homological algebra, it is difficult to draw a clear boundary between the algebraic and the geometric aspects of algebraic geometry. Our selection, to be sure, is purely conventional: On the one hand we do not include in the survey that material which could serve as a natural continuation of the previous surveys, and on the other hand, space limitations compel us to exclude such important sections of algebraic geometry as the theory of group schemes, algebraic transformation groups, Abelian varieties, the analytic theory of complex algebraic varieties (we touch upon it only incidently), the theory of automorphic functions, singularities of algebraic varieties, topology of real algebraic varieties.

Clearly, we express the general opinion if we remark that the area of algebraic geometry, relating to the theme of the present survey, is going through a period of violent growth at present, prepared to a great extent by the preceding stage of reorganization of the fundamentals of algebraic geometry. This is emphasized by the success, achieved in recent times, in solving a number of classical problems, which seemed to be unachievable earlier (irrationality of the cubic and Lurotte's problem, example of noncoincidence of homological and algebraic equivalence of cycles, Deligne's proof of Weil's conjecture on the ζ-functions of algebraic varieties).

New textbooks and surveys on algebraic geometry are [60, 123, 152, 465, 669, 733]. Conference proceedings are [134, 135, 138, 200, 722]. Memoirs and historical essays are [47, 123, 230, 258, 270, 271, 531, 532, 694, 749].

§1. Birational Geometry

This area of algebraic geometry, taking its start in the works of the German mathematicians Riemann, Clebsch, Max Noether (see the "Historical sketch" in [122]), studies the properties of algebraic varieties, which are invariant relative to birational maps. Highly effective and fruitful investigative methods were worked out during its development and an immense amount of facts was obtained, mainly for curves and surfaces. Particularly profound results were obtained for surfaces by the Italian school of algebraic geometry.

Birational geometry of varieties of dimension $n \geq 3$ upto roughly the middle of our century existed only in the form of problems and unrealized ideas, as well as of individual disconnected results not always satisfactory in the sense of rigor of proofs (see [672] and also the survey of classical results in [8]).

In recent years — in the period of the new intense flourishing of algebraic geometry — solutions have been obtained to several most important problems in birational geometry, as for example, the resolution of singularities, the generalization of the Castelnuove—Enriques contractibility criterion, the solution of Lurotte's problem, etc.

1. Models. An algebraic variety X over a field k can be examined in birational geometry in two aspects: as a support of the field of rational functions k(X) on it or as a to-

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pological space with specific properties. The birational classification of varieties coincides with the classification to within k-isomorphism of the fields of functions on them (i.e., of the arbitrary finitely generated fields over k). In the purely algebraic problem of the classification of fields the geometry emerges with the selection of a definite model, i.e., any representative from the birational class of varieties. Besides the classical models — affine and projective varieties — new ones have arisen and become widely used: abstract varieties, schemes, and algebraic spaces (see survey [31], as well as [79, 80, 81]). In addition to the fundamental papers of M. Artin, Moishezon, Knutson, the small notes [178, 252] also have been devoted to the study of algebraic spaces (or minischemes in Moishezon's definition).

The famous classical problem on the existence of a nonsingular projective model was solved in 1962 by Kironaka [116-118, 419, 420] for all dimensions in characterisitc 0 by means of the process of resolution of singularities of an arbitrary variety. Upto the time of Hironaka's work a theorem on the resolution of singularities was known for varieties of dimension $\leqslant 3$ over an arbitrary field of characteristic 0, thanks to the work of Zariski (for curves and surfaces over a complex field — this is the classical result). In characteristic $p > 0$ a theorem on resolution of singularities (and by the same token the existence of a nonsingular model) has been proved as yet only for dimensions $n \leq 3$ (for n = 2 and 3 and $p > 5$, see [128, 130]).

The important and frequently used concept of a normal variety (or normal model) was introduced by Zariski; to him is due also the procedure for the normalization of any variety X, based on a purely algebraic operation of the integral closure of local rings $O_{X,x}$, $x \in X$, in a quotient field k(X) (see [33]). A normal model does not have singularities in codimension 1; consequently, for curves this procedure leads to a nonsingular model. New results on the properties of normal varieties are related exclusively to commutative algebra and the abstract algebraic geometry (see [31]).

One of the central concepts in birational geometry is that of a minimal model. A nonsingular projective variety X over a field k is called a minimal model if every birational morphism $X \to Y$ onto a nonsingular projective variety Y is an isomorphism. However, if any birational map $X \to X$ is an isomorphism, then X is called an absolute minimal model, otherwise, a relative one. In each birational class in which a nonsingular projective model exists, a relatively minimal one exists too. An absolutely minimal model, obviously, is unique to within isomorphism in its own birational class; therefore, its existence reduces the problem of birational classification to classification within isomorphism. A nonsingular projective model has been determined uniquely in dimension 1 (see [122], for example). Enriques proved the existence of the absolutely minimal model over an algebraically closed field of characteristic 0 for all surfaces except rational and ruled ones. This result, as well as the results relating to the description of relative minimal models, have been reproduced in a number of papers [2, 68, 83, 759, 760]. In [161, 162, 181, 534, 715] they are generalized to two-dimensional regular schemes. Almost nothing is known on minimal models in dimensions $n \geqslant 3$.

Minimal models of surfaces over an algebraically closed field possess the characteristic property of the absence on them of exceptional curves of first genus (i.e., irreducible curves contractible to a nonsingular point by a birational morphism of the surface). According to the Castelnuovo—Enriques criterion (for example, see [2]) such curves E are characterized by the conditions: E is a nonsingular complete curve of genus zero and its self-intersection index equals −1 (for arbitrary regular two-dimensional schemes see [534, 715]; for contraction to a singular point see M. Artin [5], as well as [146, 147]).

The first generalizations of the Castelnuovo—Enriques criterion to the multidimensional case and only for contraction to a point were given by Kodaira in the algebraic case and by Grauert in the complex-analytic case. A general criterion was given by Moishezon [82] in the following formulation: Let $f\colon X' \to X$ be a proper morphism of complex varieties, Y' be a reduced complex subspace in X', Y be a complex subvariety, $f(Y') = Y$, $f\colon X' \setminus Y' \to X \setminus Y$ be an isomorphism, and $\dim Y' > \dim Y$; then Y' is called an exceptional subvariety of first genus, while f is called a contraction. If the number of algebraically independent meromorphic functions on X' coincides with the dimensionality of X', then the following criterion holds. In order for an irreducible complex subspace Y' of a compact variety X' to be an exceptional subvariety of first genus, it is necessary and sufficient that the following three conditions be fulfilled:

a) Y' is nonsingular, $\operatorname{codim} Y' = 1$;

b) the morphism $\varphi: Y' \to Y$ onto a complex variety exists, being a locally trivial fibering into projective spaces P^m;

c) the intersection $Y' \cdot \varphi^{-1}(y)$ of cycles on X' coincides with the class of hyperflat section into P^m, taken with a minus sign, or, what is equivalent, the restriction to Y' of the sheaf of ideals $I = O_{X'}(-Y')$ coincides with the natural very ample sheaf $O_{Y'/Y}(1)$ of the projective morphism φ.

It turned out that analogous conditions are necessary, but not always sufficient, for contraction in the category of algebraic varieties [82]. This fact serves as one of the reasons for the origin of the concept of minischeme or algebraic space for which the contractibility criterion now acts with full force (Moishezon [82], Artin [142]). Artin [142] studies the more general situation of arbitrary modification in algebraic spaces.

Lascu's papers [525-527] also were devoted to the generalization of the Castelnuovo–Enriques criterion to n-dimensional varieties. His contractibility criterion in the category of algebraic varieties contains, besides conditions a)-c), the further condition $R^1 f_* I^n = 0$ for $n \geq 2$, where $I = O_{X'}(-Y')$ is a sheaf of ideals, setting Y' into X'. In [45] Moishezon's contractibility criterion is extended to arbitrary complex varieties.

Hironaka [421] constructed an example of a birational morphism of three-dimensional varieties, not decomposable into a composition of monoidal transformations with nonsingular centers (see below), from which it follows that the absence on a variety of an exceptional subvariety in the sense of Moishezon still does not signify that it is minimal.

The concept of a minimal model was extended to families of algebraic curves and Abelian varieties (see §4).

The traditional approach to the concept of a "good" model of an algebraic variety consists in selecting a "good" birational immersion of it into a projective space, as well as into other special varieties. For example, the fundamental model of an n-dimensional variety X in Italian geometry was its birational representation as a hypersurface in P^{n+1} with the usual singularities (see [672, 757]). The fact that various good representations are possible for an algebraic curve: as for example, as a planar curve having, possibly, only the usual double singular points; imbedding as a nonsingular curve in P^3; canonic immersion of a nonhyperelliptic curve of genus $g > 1$ into P^{g-1}; representation as a finite (ramified) covering of the straight line P^1; the immersion into it of a Jacobi variety; is used for investigating its properties (see §7) and particularly for studying the moduli space (see §5).

See §7 for various representations of surfaces.

The idea of constructing a model in birationally invariant terms, i.e., in terms of the field of functions $K = k(X)$, is due to Zariski. In dimensions greater than 1 such a model is not an algebraic variety in the usual sense (it is too "endless"), namely, it is the so-called Zariski–Riemann space (see [33]); its set of points consists of all possible regular local subrings $O \subset K$ of maximal dimension, containing field k. In the one-dimensional case it coincides with the usual nonsingular complete model. In dimension $n = 2$ its description is reproduced in Lipman's addendum in book [757]; it is a certain formalization of the classical conception of "infinitely close points" and of linear systems with prescribed base conditions, which were the fundamental instrument in the birational geometry of M. Noether and of the Italian geometers (see [752]). An interesting variant of this two-dimensional model was given by Manin [60], called a "foamy space." The birational invariance of a "foamy space" and especially its Picard groups were used in [60] (also see [68, 36-38]) for a birational classification of rational surfaces over perfect fields, as well as for the study of groups of birational automorphisms.

2. Birational Maps and Birational Invariants. The simplest birational maps are the quadratic (invertible) transformations of projective spaces. They played a very important role in classical birational geometry, mainly because they generate the whole group of birational automorphisms on the plane P^2 over an algebraically closed field (Noether's theorem, see Para. 3 below). Certain concrete examples of quadratic transformations are met with in the literature and here (for example, see [198, 203, 407, 618]). However, in modern birational geometry the central place is occupied by the (more elementary and most important) concept of a monoidal transformation with a nonsingular center, also called a blowing-up or a σ-

process. This concept is a special case of the most general concept of the blowing-up of a coherent sheaf of ideals.

Let X be an arbitrary algebraic variety (or scheme), I be a coherent sheaf of ideals in O_X, $Y \subset X$ be a closed subscheme given by I. The morphism f: $X' \to X$ is called a blowing-up of sheaf I or a monoidal transformation with center in Y if the following conditions are fulfilled:

1) $f^*(I) = I \underset{O_X}{\otimes} O_{X'}$ is an invertible sheaf of ideals on X';

2) if g: $X'' \to X$ is an arbitrary morphism satisfying condition 1), then there exists a unique morphism h: $X'' \to X'$ with $f \circ h = g$ (see Hironaka [116], and also [60, 61]).

A monoidal transformation is a proper birational morphism "blowing up" Y upto the divisor $f^{-1}(Y) = Y'$ in X' and being an isomorphism outside Y'. Its explicit construction has the form $f: \mathrm{Proj}\left(\bigoplus_{k=0}^{\infty} I^k\right) \to X$; if X and Y are smooth and codim Y = $r \geqslant 2$, then X' and $f^{-1}(Y) = Y'$ also is smooth and $Y' \to Y$ is a locally trivial fibering into projective spaces, being a projectivization of a normal vector bundle $N_{Y/X}$ (see [60, 122]). By virtue of property 2) the morphism of contraction of an exceptional divisor of first genus (if it exists) is a monoidal transformation.

With the aid of monoidal transformations with nonsingular centers in smooth varieties Zariski, Hironaka, and Abhyankar resolved the singularities of the singular varieties embedded in them and proved a theorem on the removal of points of indeterminacy of rational maps or the trivialization of coherent sheafs of ideals in the sense of Hironaka [116]. We are dealing with the following assertion. Let g: $X \to Y$ be an arbitrary rational map of a nonsingular variety X into a variety Y; then there exist a finite chain of monoidal transformations with nonsingular centers $f_i: X_{i+1} \to X_i$, $i = 0, \ldots, N-1$, $X_0 = X$, and a birational morphism $h: X_N \to X$, such that $g = h \circ (f_1 \circ \ldots \circ f_N)^{-1}$. Hironaka [116-118] proved this fact for varieties of arbitrary dimension in characteristic 0 (it had been known earlier for surfaces and for three-dimensional varieties) and Abhyankar [128, 130] proved it for varieties of dimension $\leqslant 3$ in characteristic $p > 0$.

Any vector bundle E over a nonsingular projective variety X in characteristic 0 can be turned into an extension of linear bundles on some projective nonsingular variety X' obtained from X by means of a finite sequence of monoidal transformations with nonsingular centers [116]. A flattening procedure for nonflat morphisms is effected with the aid of monoidal transformations of the base ([653], and also Hironaka (preprint)).

From nonsingular projective surfaces every birational morphism is a composition of monoidal transformations with centers at points and every rational map can be decomposed (essential uniquely) into monoidal transformations and transformations inverse to them [534, 715]. The already-mentioned example of Hironaka shows that in the three-dimensional case not every birational morphism admits of a decomposition into a composition of monoidal transformations. The representation of any such morphism in the form of a sequence of blowing-ups and contractions with nonsingular centers remains an unsolved problem.

From the theorem on the removal of points of indeterminacy of rational maps in dimension n we can derive a theorem on the birational invariance of the cohomology spaces $H^i(X, O_X)$ for smooth complete varieties X (Hironaka [116], Abhyankar [129]) and, in particular, we can give an affirmative answer to the classical question on the birational equivalence of arithmetic genus

$$p_a(X) = \sum_{i=0}^{n-1} (-1)^i \dim_k H^{n-i}(X, O_X)$$

(for all n in characteristic 0 and for $n \leqslant 3$ in characteristic $p > 5$). By Serre duality follows hence the birational invariance of $H^{n-i}(X, \Omega_X^n)$, where Ω_X^r denotes a pencil of regular differential r-forms on X; when r = n this is the canonic invertible pencil on X, often denoted also as ω_X. The birational invariance of geometric genus $p_g = \dim_k H^n(X, O_X) = \dim_k H^0(X, \omega_X)$, as well as of all spaces of regular differential forms $H^0(X, \Omega_X^r)$, $r \geqslant 1$, was known earlier for all smooth varieties in any characteristic (see [123], for example). Among the invariants connected with differential forms the ones most commonly used are the so-called multiple genera $P_m = \dim_k H^0(X, \omega_X^{\otimes m})$, $m \geqslant 2$ (see [757, 455]). Paper [13] was devoted to the study of various tensor invariants arising from differential forms.

An important birational invariant, connected with multiply canonic maps of X into projective spaces, is the so-called Kodaira dimension $\varkappa(X)$, defined as follows. Let $\varphi_m: X \to P^N$ be a rational map defined by the invertible pencil $\omega_X^{\otimes m}$, $m \geqslant 1$, where $N = \dim_k H^0(X, \omega_X^{\otimes m}) - 1$; then

$$\varkappa(X) = \max_m (\dim \varphi_m(X))$$

for at least one m if $N \geqslant 1$, otherwise, $\varkappa(X) = -\infty$. Some applications of the invariant $\varkappa(X)$ for the study of varieties of dimension $n \geqslant 3$ are contained in [737, 455]. Invariants, connected with the Kodaira dimension, and their application to the problem of the rationality of three-dimensional varieties occur in Iitaka [460].

The classical results on the birational invariance of the fundamental group on smooth projective varieties over a complex field were generalized by Grothendieck (see survey [31]). Another simpler invariant can be extracted from the fundamental group, viz., the torsion subgroup in the group of one-dimensional homologies or (from the formula of universal coefficients) the torsion subgroup in two-dimensional cohomologies with integral (or integral l-adic in characteristic $p \neq l$) coefficients. By Poincaré duality this invariant coincides with the torsion subgroup in the Néron–Severi group of divisor classes. Torsions in the latter group were known in the classical literature as Severi "divisors" and were frequently used for constructing irrational varieties with zero genera (see Roth [672]). In 1959 Serre [706] proved that the fundamental group (and, consequently, the Severi "divisors") does not distinguish unirrational and rational varieties, having shown by the same token the inconsistency of all the classical attempts to use this invariant to construct counterexamples to Lurotte's problem in dimension $n \geqslant 3$.

Such counterexamples were constructed independently (and practically simultaneously) by Clemens and Griffiths [205] (n = 3), Iskovskikh and Manin [39] (n = 3), and Artin and Mumford [143] (for any n). Let us describe the invariants they used.

Artin and Mumford used torsions in three-dimensional cohomologies $\mathrm{Tors}\, H^3(H, Z_l)$. The birational invariance of this group can be established directly by use of the theorem on the removal of points of indeterminacy of a rational map [143]; however, this fact was first discovered by Grothendieck (and even without the resolution theorem) in [402]. He proved the birational invariance of the Brauer group Br(X) and established its connection with Tors $H^3(X, Z_l)$. Deligne's report [221] was devoted to an exposition of Artin and Mumford's work. For surfaces over a nonclosed field the Brauer group is closely related with another birational cohomological invariant $H^1(k, \mathrm{Pic}\, X)$ (Galois cohomologies) introduced earlier by Manin [68] (also see [60]).

A new interesting birational invariant — the polarized intermediate Jacobian $J_3(X)$ — was found by Clemens and Griffiths [205]; with its help they proved the irrationality of a three-dimensional cubic hypersurface in $\mathbf{P}^4$ over a complex field, having by the same token solved a famous classical problem. For smooth projective three-dimensional varieties X with $H^0(X, \omega_X) = 0$ over the complex number field $\mathbf{C}$ the intermediate Jacobian $J_3(X)$, defined as the complex torus $H^{1,2}(X)/H^3(X, Z)$, where $H^{1,2}(X)$ is the corresponding Hodge component in the expansion of $H^3(X, C)$, is an Abelian variety. If $H^1(X, Z) = 0$, then a skew-symmetric form on $H^3(X, Z)$ (the intersection index) prescribes on $J_3(X)$ a certain principal polarization Θ. Under birational transformations only the Jacobians of curves with the natural Poincaré polarization can be appended to $[J_3(X), \Theta]$; therefore, if $[J_3(X), \Theta]$ is not the Jacobian of any curve whatsoever, equipped with Poincaré polarization, then X is an irrational variety. The fact that for a cubic X the five-dimensional polarized Abelian variety $[J_3(X), \Theta]$ is not the Jacobian of a curve can be established by studying the geometric properties of the divisor Θ (for example, its singularities are not the same as in the Poincaré divisor). This is a special case of the so-called Prim varieties. A good geometric exposition of this circle of questions appears in Tyurin [108]. A generalization in characteristic $p > 0$ was given by Murre [594].

Iskovskikh and Manin [39] used a generalization to the three-dimensional case of the birationally invariant conception of "virtual linear systems with prescribed base conditions" which was widely used by Noether and the Italian geometers in the theory of surfaces. In the modern interpretation this is a certain theory of intersections (or generalized Chow rings) on Zariski–Riemann spaces. Birational invariants arise here as certain inequalities connected, on the one hand, with the behavior of canonic classes under birational transformations [639] and, on the other, with the purely geometric properties of positiveness and mobility of cycles. A property was proved in [39] that on a smooth three-dimensional hypersurface X of degree 4 in P^4 there are no birational automorphisms besides the projective ones and, consequently, X is not rational. The unirationality of certain such hypersurfaces was established earlier by Segre [695]. The original idea of the proof and even the formulation of the result are due to Fano who, however, was unable to give an exhaustive proof.

Individual special results on rational and unirational varieties are to be found in [73, 74, 125, 234, 264, 442, 470, 659-662, 727, 743].

See [98] for certain other birational invariants connected with the concept of the rational equivalence of 0-cycles.

3. Cremona Transformations and Other Birational Automorphisms. The group of birational automorphisms of an n-dimensional projective space P^n is called an n-dimensional Cremona group and is denoted by Cr_n. The already-mentioned Noether theorem asserts that the two-dimensional Cremona group over an algebraically closed field is generated by quadratic transformations (or by the one standard transformation $(x_0, x_1, x_2)\to(x_0^{-1}, x_1^{-1}, x_2^{-1})$ and projective transformations). It is re-proved in [2, 84, 670] by essentially the same method. The classical results of Cantor and Wiman on the classification to within conjugacy of finite subgroups (in particular, of involutions, i.e., subgroups of order 2) are partially reestablished in [36, 37, 68, 69]. Individual results on quadratic transformations of a plane are obtained in [199, 235]. Certain concrete examples of Cremona transformations in a plane are considered in [102, 201, 251, 522].

Interesting results on groups of birational surfaces of del Pezzo over a perfect field, in particular, cubic surfaces in P^3, were obtained by Manin [60, 67, 68]. He has described not only the generator but also all the relations in the cases investigated. In [60] birational automorphisms of cubic hypersurfaces are applied to the study of the rational points on them.

Birational automorphisms of rational surfaces with a pencil of rational curves defined over a perfect field were studied in [36, 37].

Almost nothing is known about groups of birational automorphisms of varieties of dimension $n\geqslant 3$ except for individual results on the Cremona group and on certain concrete Cremona tranformations.

The most essential results in this direction were obtained by Demazure [227] (also see [423]). He generalizes the well-known result of Enriques on maximal algebraic subgroups in a two-dimensional Cremona group to dimensions greater than 2. Enriques' result is that any connected maximal algebraic subgroup in Cr_2 is a group of biregular automorphisms on some relatively minimal model of field k(x, y) (i.e., on a plane or one of the ruled surfaces F_n, $n\geqslant 0$, $n\neq 1$). Demazure studies connected algebraic subgroups in Cr_n, containing an n-dimensional decomposable torus. Their classification is reduced to the classification of the so-called "Enriques systems" (analogs of root systems). Such groups act on certain varieties which are compactifications of the n-dimensional torus. A complete classification can be obtained for semisimple groups.

Frumkin [115] studies the classical concept of the genus of a birational automorphism of a three-dimensional variety, i.e., the maximal value of the genera of its base curves. Transformations of genus $\leqslant g$ form a subgroup in the whole automorphism group. It is shown that in Cr_3 corresponding filtration is infinite; in particular, Cr_3 is not generated by quadratic transformations (even by any transformations of bounded genus). An interesting fact is established: any birational map of complete smooth three-dimensional varieties in characteristic 0 can be decomposed, "modulo transformation of genus 0," into a composition of monoidal transformations and transformations inverse to them.

Certain multidimensional quadratic transformations are studied in [198, 203, 618].

It is well known that a Cremona transformation of a plane and its inverse are of like degree. This is already not so in dimension 3. Certain special transformations of such type are studied in [702-704]. Certain other examples of birational transformations are in [183, 198, 544, 569, 692].

In conclusion we note that one of the most interesting unsolved problems is the problem of describing the groups of birational automorphisms of three-dimensional Fano varieties, in particular, the three-dimensional cubic.

The problem of characterizing unirational and rational varieties occupies a central position in the birational classification of three-dimensional varieties.

§2. Riemann–Roch Theorem and Intersection Theory

1. Riemann–Roch Theorem. This theorem, in the classical formulation computing the dimension of a complete linear system of divisors on curves or surfaces, was generalized in 1956 to varieties of arbitrary dimension by Hirzebruch (see [119]) (in the complex case). In 1957 Grothendieck gave a purely algebraic proof of this theorem, true for nonsingular projective varieties over an algebraically closed field of arbitrary characteristic [175, 397]. The Riemann–Roch–Grothendieck theorem asserts the realizability of the relation

$$f_*(\mathrm{ch}_X(\mathscr{F})\,\mathrm{Todd}(T_X))=\mathrm{ch}_Y(f_*(\mathrm{cl}(\mathscr{F})))\cdot\mathrm{Todd}(T_Y)$$

for any proper morphism $f:X\to Y$ of smooth quasiprojective varieties and of a coherent sheaf $\mathscr{F}$ on X. The values on both sides belong to $A(Y)\otimes\mathbf{Q}$, where A(Y) is the Chow ring of variety Y; here f_* is determined with the aid of the operation of "direct image of a cycle," cl $(\mathscr{F})$ is the class of $\mathscr{F}$ in the Grothendieck group $K_{\mathrm{Coh}}(X)$ of the category of coherent sheafs on X, $f_*(\mathrm{cl}(\mathscr{F}))=\sum_i(-1)^i\mathrm{cl}(R^if_*\mathscr{F})$, $\mathrm{ch}_X(-)$, $\mathrm{ch}_Y(-)$ [respectively, Todd()] are universal polynomials with coefficients in Q of the Chern classes of sheaf $\mathscr{F}$ (respectively, of the Chern classes of the tangent bundles T_X, T_Y to varieties X and Y). The Chern classes $c_i(E)\in A^i(X)$ of a locally free sheaf E were defined by Grothendieck in [394] with the aid of the splitting principle. For a coherent sheaf they are determined by additivity, using the equality $K_{\mathrm{Coh}}(X) = K(X)$, where K(X) is the Grothendieck group of the category of locally free sheaves on variety X.

When Y is a point the preceding formula turns into the equality

$$\chi(Y,\mathscr{F})=\sum(-1)^i\dim_k H^i(X,\mathscr{F})=[\mathrm{ch}_X(\mathscr{F})\cdot\mathrm{Todd}(X)](X), \qquad (*)$$

where Z[X] denotes the n-dimensional component for any $Z\in A(X)\otimes\mathbf{Q}$ [$A^n(X)$ is identified with Z]. When k = C and $\mathscr{F}$ is a locally free sheaf equality (*) turns into Hirzebruch's theorem. Here, instead of the ring A(X) we examine the ring of rational cohomologies H*(X, Q). In the general case there also exist modifications of the Riemann–Roch–Grothendieck formula, in which the Chern classes take values in l-adic or crystal cohomologies (see [157-159, 396]).

To Grothendieck is due the idea of defining Chern classes without using either cycles or cohomologies (the 1957 manuscript was published in [397]). In this theory the ring $A(X)\otimes\mathbf{Q}$ is replaced by the ring $\mathrm{Gr}^{\cdot}_{\mathrm{top}}(X)$, i.e., a graded ring relative to the descending filtration $K_{\mathrm{Coh}}(X)=F^0_{\mathrm{top}}\supset\ldots\supset F^n_{\mathrm{top}}$ defined by the codimension of the sheaf's support. The Chern classes are defined with the aid of the formula $c_i(\mathscr{F})=\gamma^i(x-\varepsilon(x)) \bmod F^{i+1}_{\mathrm{top}}$, where $x=\mathrm{cl}(\mathscr{F})\in K_{\mathrm{Coh}}(X)$, $\varepsilon:K_{\mathrm{Coh}}(X)\to\mathbf{Z}$ is an augmentation, γ^i are operations of exterior power, extendable from ring K(X) onto $K_{\mathrm{Coh}}(X)$.

If f is a morphism of a regular embedding with a conormal sheaf N, then the Riemann–Roch–Grothendieck formula yields, to within torsion, an expression for the Chern classes $c_i[f_*(x)]$ of the form $c_i(f_*(x))=f_*(P_i((c_\alpha(x))_\alpha,(c_\beta(N))_\beta$, where the P_i are certain universal integral polynomials.

In [466] this formula is proved without an assumption on torsion (the "Riemann–Roch theorem without denominators"). In 1957 Grothendieck [397] proved (in the case char k = 0) such a formula (without torsion) using the ring $\mathrm{Gr}^{\cdot}_{\mathrm{top}}$, in one variant of the theory.

The Riemann–Roch theorem is generalized in [699] to the case of a proper morphism of a locally complete intersection of Noether schemes possessing ample invertible sheaves. In order to give meaning to the Riemann–Roch formula in this case, a ring $\mathrm{Gr}^{\cdot}(X)$ is introduced, serving as a replacement of ring A(X) or $\mathrm{Gr}^{\cdot}_{\mathrm{top}}(X)$ (the absence of the ground field and of the nonsingularity condition). This ring is a graded ring relative to λ-filtration in K(X), in which $\mathrm{Filt}^i K(X)$ is generated by expressions of the form $\gamma^{i_1}(x_1-\varepsilon(x_1))\cdot\ldots\cdot\gamma^{i_k}(x_k-\varepsilon(x_k))$, $i_1+\ldots+i_k\geqslant i$. The Riemann–Roch theorem for a projective morphism of smooth algebraic varieties, using ring $\mathrm{Gr}^{\cdot}(X)$ instead of A(X), is set forth in [61]. The absence of the condition of regularity of the schemes does not permit the identification of rings $K_{\mathrm{Coh}}(X)$ and K(X), which compels Grothendieck to examine only the ring K(X) (for which alone the operations γ^i are defined). The homomorphism f_*: $K(X)\to K(Y)$ is determined with the aid of the techniques of derived categories. In the case of smooth quasiprojective schemes the rings $\mathrm{Gr}^{\cdot}(X)$, $\mathrm{Gr}^{\cdot}_{\mathrm{top}}(X)$ and A(X) are isomorphic modulo torsion.

Subsequent generalizations and variants of the Riemann-Roch theorem are discussed in Reports 0 and XIV of seminar [699]. The Lefschetz formula on the fixed points of coherent sheaves is derived in [232] from the Riemann–Roch formula. For the case k = **C** an analytic proof of this theorem (for locally free sheaves) was given by Atiyah and Singer (see [7]).

2. Computation of K(X) and of the Chern Classes. A complete computation of the Grothendieck ring K(X) for the category of locally free sheaves on a scheme X is known only in very special cases. Obviously, this problem is very difficult since the knowledge of K(X) permits in principle the computation of the corresponding graded ring $\mathrm{Gr}^{\cdot}(X)$ and, together with that (if X is a quasiprojective smooth scheme), of ring $A(X)\otimes\mathbf{Q}$. The computation of the latter ring has for a long time now been one of the most difficult problems in algebraic geometry (see §3).

If X is a Noetherian scheme, $i: X'\hookrightarrow X$ is a closed subscheme of it, $j: U\hookrightarrow X$ is an embedding of the complement to X', then there holds the exact "homotopy sequence":

$$K(X')\xrightarrow{i_*}K(X)\xrightarrow{j^*}K(U)\to 0.$$

This statement (see [175, 699]) plays an important technical role in the computation of K(X).

If (X_i, f_{ij}) is a projective system of Noetherian schemes, while the f_{ij} are flat, then there exists the isomorphism $K(X)\simeq\varinjlim K(X_i)$, where $X=\varprojlim(X_i, f_{ij})$ ([699], Rept. IV). The group K(X) changes under an extension of the ground field k'/k. The nucleus of the homomorphism $K(X)\to K(X_{k'})$ is a periodic group, it is trivial if k is separably closed, while k'/k is a separable extension (see [699], Rept. XI).

The homomorphism $L\to c_1(L)$ defines the isomorphism of the Picard group Pic(X) and of the $\mathrm{Gr}^1(X)$. If X is a regular scheme with a very ample sheaf, then $K(X)\simeq K_{\mathrm{Coh}}(X)$ and the isomorphism

$$K_{\mathrm{Coh}}(X)/F^2_{\mathrm{top}}(X)\simeq\mathbf{Z}\oplus\mathrm{Pic}(X)$$

holds ([61]; [699], Rept. X).

If X is a vector bundle corresponding to a locally free sheaf E of finite type on a Noetherian scheme S, then the homomorphism f^*: $K(S)\to K(X)$ is an isomorphism ([699], Rept. IX). In particular, if X is a variety over field k, then $K(X\times\mathbf{A}^n_k)\simeq K(X)$ ([175]; [699], Rept. 0).

In [699], Rept. IX, K(X) is computed, where X is the principal homogeneous space for the torus $G^n_{m,S}$ and for the general linear $GL(n)_S$. Group K(X) for splittable reductive group schemes was studied by Serre [708].

An important role in the proof of the Riemann–Roch theorem is played by the computation of K(X), where X = **P**(E) is a projectivization of the vector bundle defined by a locally free sheaf E of rank r on a regular connected Noetherian scheme S with an ample sheaf. The ring K[**P**(E)], treated as a K(S)-algebra relative to the isomorphism $f^*: K(S)\to K[\mathbf{P}(E)]$, is generated by the element $e = \mathrm{cl}[\mathcal{O}_X(1)]$, while the ideal of the relations is generated by the element $\sum_{i=0}^{r+1}(-1)^i\,\mathrm{cl}\,(\overset{r+1-i}{\wedge}E)\,e^i$. In particular, K[**P**(E)] is a free K(S)-module of rank r+1. If S is a point, we obtain hence that $K(\mathbf{P}^r_k)\simeq\mathbf{Z}[T]/(1-T)^{r+1}$ (T corresponds to cl $(\mathcal{O}_{\mathbf{P}^r_k}(1))$). For the trivial bundle on the k-scheme S we have $K(S\times\mathbf{P}^n_k)\simeq K(S)\otimes K(\mathbf{P}^n_k)$. When k = **C**, n = 1, this fact is analogous to the theorem of complex Bott periodicity in topological K-theory.

The computation of K(X), where X is a flag variety (in particular, a Grassmann variety) (see [699], Rept. VI), serves as a generalization of the preceding results.

K(X'), where X' is the result of applying a monoidal transformation to a smooth projective k-scheme X with center in a regularly embedded subscheme Y, is studied in [61], [699], Rept. VII, and [175]. The group K(X') is computed in terms of K(X), K(Y), and the class of the conormal sheaf $N_{Y/X}$ and its exterior powers.

The Chern classes $c_i(X')$ were computed by complex methods by Porteous [639]. In case $\operatorname{char} k > 0$ his formula can be derived from the Riemann–Roch–Grothendieck theorem (see [148, 262] for other proofs of it).

Kleiman [488] obtained important results on Chern classes of sufficiently ample vector bundles E on a quasiprojective smooth variety X [for which E(−1) is generated by global sections]. Using the immersion of X with the aid of E into a Grassmann variety and a detailed study of its Schubert subvarieties, he showed that the Chern classes $c_i(E)$ of such bundles generate the ring $A(X)\otimes\mathbf{Q}$. In addition, $c_i(E)$ can be represented by a certain subvariety Z_i which is nonsingular for i = 1 or $i > (\dim X-2)/2$. A detailed description of the singularity Z_i is given in the general case (see [492, 493]) (obtained independently by Griffiths as well).

See the survey [31] on the Chern classes of ample bundles in the sense of Hartshorne.

Papers [261, 688-691] are devoted to ascertaining the connection between the Chern–Grothendieck characteristic classes and the classical definitions of covariant and canonic Todd–Segre sequences.

The connection between algebraic and topological K-theories was clarified in [467].

3. Intersection Theory. For nonsingular projective varieties this theory was constructed by van der Waerden, Chevalley, Samuel, Weil (see [8] for a history of the question). The theory of Chow rings is set forth in Chevalley's seminar [698]. For the case of divisors an elementary proof of the properties of the intersection index is contained in [123]. Papers [165, 168, 663, 667, 748] are devoted to certain special aspects of intersection theory.

An interesting problem in intersection theory is the problem of generalizing it to a wider class of schemes.

In Shafarevich's lectures [715] (also see [218, 534, 609]) intersection theory is constructed for two-dimensional regular schemes, proper over a normal local ring. This theory plays an important role in the classification of degenerate fibers (see §4) and in the study of the singularities of surfaces. Intersection theory on two-dimensional Zariski spaces (see §1) is set forth in [60, 218, 757]. Height theory permits us to determine the intersection of divisors on arithmetic surfaces (regular two-dimensional schemes, proper over the integer ring of the field of algebraic numbers) [611]. An interesting global version of this theory, accounting for Archimedean valuations, and a generalization of the connection formula to this case have been given by Arakelov (unpublished).

In the case of singular normal surfaces three different methods were proposed for the definition of the local intersection index: topological [656, 657], using resolution of singularities [54], and purely algebraic [22]. All these definitions coincide. We remark that, in general, the intersection index in this case is a rational number.

An absolutely original idea for the generalization of intersection theory to arbitrary schemes belongs to Grothendieck [699]. Instead of the Chow ring A(X) he proposed to examine the ring Gr.(X), i.e., a graded ring corresponding to the filtration $F_i(X)=\{\mathrm{cl}(\mathscr{F})|\dim\operatorname{supp}\mathscr{F}\leqslant i\}$ of the Grothendieck ring $K_{\mathrm{Coh}}(X)$ of coherent sheaves on scheme X. This ring plays the role of the integral algebraic homology classes. The role of the cohomologies is played by the ring $\mathrm{Gr}^{\cdot}(X)$ figuring in the Riemann-Roch-Grothendieck theorem (see Para. 1). There holds the multiplication formula $\mathrm{Filt}^i K(X)\cdot F_j(X)\to F_{j-i}(X)$, induced by the tensor product operation for sheaves, giving the structure of a $\mathrm{Gr}^{\cdot}(X)$-module on Gr.(X). If X is a proper scheme over a field k, then a degree homomorphism $\deg: G_0(X)\to\mathbf{Z}$, exists which determines the pairing $\mathrm{Gr}^i(X)\times\mathrm{Gr}_i(X)\to\mathbf{Z}$ (the intersection index). In the case of smooth schemes over a field the rings $\mathrm{Gr}^{\cdot}(X)$ and Gr.(X) are isomorphic ("Poincaré duality"). In addition, if, further, X is quasiprojective, then to within torsion these rings are isomorphic with the Chow ring A(X) (in the general case only the equality $\operatorname{Pic}(X)\simeq\mathrm{Gr}^1(X)$ holds) and we obtain the usual intersection theory.

§3. Algebraic Cycles

1. Equivalence Relations on Algebraic Cycles. An algebraic cycle on a scheme X is a formal linear integral combination $Z = \Sigma m_i Z_i$ of closed integral subschemes Z_i. The set of all cycles forms an Abelian group $C^*(X) = \bigoplus_p C^p(X)$, graded with respect to codimension. This group has been studied, mainly, as yet only in the case when X is a smooth quasiprojective variety of dimension n over an algebraically closed field k (only the component $C^1(X)$, closely connected with the Picard group of scheme X, has been studied in the general case). In what follows we examine this case alone. When cycles Z and W intersect "tamely" the methods of local algebra permit us to determine the intersection cycle $Z\cdot W = \Sigma i(Z, W, Y_i) Y_i$ [100, 705] possessing the properties of commutativity and associativity.

Cycles Z_1 and Z_2 on X are said to be algebraically equivalent if there exist a quasi-projective connected variety T and a cycle Z on $X \times T$ such that for closed points $t_1, t_2 \in T$ the cycle $Z\cdot(X \times t_1)$ [respectively, $Z\cdot(X \times t_2)$] is defined and coincides with Z_1 (respectively, with Z_2). If in this definition T can be taken as a rational variety, then Z_1 and Z_2 are said to be rationally equivalent. When $Z_1, Z_2 \in C^1(X)$, rational equivalence coincides with the linear equivalence of divisors.

With the aid of the "shift lemma" the intersection operation allows us to define the multiplication of classes with respect to rational equivalence, converting the corresponding factor group into a commutative associative ring denoted by A(X) and called the Chow ring of variety X. For any morphism $f: X \to Y$ of algebraic varieties there have been defined the functorial homomorphism of rings $f^*: A(Y) \to A(X)$ and the homomorphism of group $f_*: A(X) \to A(Y)$ related by the projection formula $f_*(f^*(W)\cdot Z) = W\cdot f_*(Z)$. A detailed account of this theory is contained in Chevalley's seminar [698] (also see [436]).

An equivalence relation R on $C^*(X)$, cruder than rational equivalence, for which the corresponding factor set $C^*(X)/R$ is a ring relative to the operations of multiplication and intersection of cycles with functional homomorphisms f^* and f_* [everything is consistent with the canonic homomorphism $A(X) \to C^*(X)/R$], is called an adequate equivalence relation.

Examples of adequate equivalence relations are algebraic equivalence, τ-equivalence ($Z_1 \underset{\tau}{\sim} Z_2$, if $\exists m \neq 0$ such that $mZ_1 \underset{\text{alg}}{\sim} mZ_2$), numerical equivalence ($Z_1 \underset{\text{num}}{\sim} Z_2$, if for any cycle W of complementary codimension, $Z\cdot W_1 \underset{\text{alg}}{\sim} Z\cdot W_2$, if both sides are defined). For any theory of Weil cohomologies $H^*(X)$ (see [31, 485]) we can define the adequate homological equivalence: $Z_1 \underset{\text{hom}}{\sim} Z_2$, if $\gamma(Z_1) = \gamma(Z_2)$, where $\gamma: C^*(X) \to H^{2*}(X)$ is a map of cycles. The corresponding quotient-ring $C^*(X)/\text{hom}$ can be identified using map γ with the subring $H^*_{\text{alg}}(X) \subset H^{2*}(X)$ of algebraic cohomology classes. It is not known whether $H^*_{\text{alg}}(X)$ depends upon the cohomology theory chosen.

Griffiths [388] examined an interesting adequate equivalence relation. For any q-dimensional cycle Z on a variety X and for an algebraic family $W \subset X \times T$ of (q − 1)dimensional cycles on X, he defines an incidence divisor $D_Z \subset T$, by setting $D_Z = \text{np}_T W\cdot(Z \times T)$. A cycle Z is said to be incidently equivalent to zero if for all T and W the divisor D_Z is linearly equivalent to zero.

Important adequate equivalence relations are connected with maps of cycles in Abelian varieties. For example, if k = C and $J^r(X)$ is the r-th intermediate Weil Jacobian of the variety X, then there is defined the canonic homomorphism $\omega: C^r_{\text{hom}}(V) \to J^r(X)$ [where $C^r_{\text{hom}}(X)$ is the group of cycles, homologous to zero, of codimension r on X]. The nucleus of this homomorphism defines the Weil equivalence relation [487]. An analogous construction, using the Griffiths Jacobian, defines the Griffiths equivalence relation. Cycles Z and Z' are said to be Abel equivalent, $Z \underset{\text{abl}}{\sim} Z'$, if Z − Z' lies in the nucleus of any rational homomorphism of X into an Abelian variety [487].

Let α denote one of the equivalencies introduced above and let $C^*_\alpha(X) = \bigoplus_p C^p_\alpha(X)$ be the corresponding subgroup of cycles α-equivalent to zero.

There hold the embeddings of graded groups

$$C^*_{\mathrm{rat}}(X) \subset C^*_{\mathrm{inc}}(X) \subset C^*_{\mathrm{alg}}(X) \subset C^*_\tau(X) \subset C^*_{\mathrm{hom}}(X) \subset C^*_{\mathrm{num}}(X) \subset C^*(X).$$

For one-dimensional components (i.e., for divisors) there hold the equalities $C^1_{\mathrm{rat}}(X) = C^1_{\mathrm{inc}}(X)$ [388] and $C^1_\tau(X) = C^1_{\mathrm{num}}(X)$. The factor group $C^1_{\mathrm{alg}}(X)/C^1_{\mathrm{rat}}(X)$ has the structure of an Abelian variety [the Picard variety $\mathrm{Pic}^\circ(X)$ of variety X], the factor group $C^1(X)/C^1_{\mathrm{alg}}(X)$ is finitely generated [the Néron—Severi group NS(X) of variety X], while its subgroup $C^1_\tau(X)/C^1_{\mathrm{alg}}(X)$ is finite (this last result is very well known; e.g., see [8]).

The appearance of Griffiths' counterexample to the expected equality $C^*_\tau(X) = C^*_{\mathrm{hom}}(X)$ caused a sensation [380]. An analogous example for an l-adic homological equivalence was next constructed by Grothendieck in arbitrary characteristic [473]. The latter construction makes essential use of the Picard—Lefschetz theory (see §4).

The question of the coincidence of $C^*_{\mathrm{num}}(X)$ and $C^*_{\mathrm{hom}}(X)$ is still open (see later on).

The group $C^*_{\mathrm{alg}}(X)/C^*_{\mathrm{rat}}(X)$ is not "finite dimensional" (the precise meaning of this is discussed in [56, 98, 487]). For example, for algebraic surfaces with $p_g > 0$ the cosets in $C^n_{\mathrm{alg}}(X)/C^n_{\mathrm{rat}}$ do not form an algebraic family [56]. Conversely, the factor group $C^r_{\mathrm{alg}}(X)/C^r_{\mathrm{inc}}(X)$ can be rigged with the structure of an Abelian variety [Griffiths' r-th Picard variety $\mathrm{Pic}^\circ_r(X)$] [388].

It is not known whether the group $C^*_\tau(X)/C^*_{\mathrm{alg}}(X)$ is finite. The group $C^r_{\mathrm{hom}}(X)/C^r_\tau(X)$ [called the Griffiths group in [473] and denoted $\mathrm{Griff}^r(X)$] is assumed to be finitely generated (see [388]). A closed connection is shown in [473] of the Griffiths group of common fiber of the Lefschetz sheaf on a variety X with the group of primitive algebraic classes of cohomologies of X. We do not know, for example, whether $\mathrm{Griff}^r(X)$ is a birational invariant of variety X.

The group $C^*(X)/C^*_{\mathrm{hom}}(X)$ is isomorphic with the subgroup of algebraic classes of cohomologies $H^*_{\mathrm{alg}}(X)$ and, therefore, is finitely generated for any theory of Weil cohomologies. In particular, the group $C^*(X)/C^*_{\mathrm{num}}(X)$ is finitely generated (independently of the choice of cohomologies!). This fact was proved by Grothendieck (using l-adic cohomologies) and by Luckin [538] (using his own cohomologies).

Griffiths [388] proves the embedding $C^r_{\mathrm{alg}}(X) \cap C^r_{\mathrm{Griff}}(X) \subset C^r_{\mathrm{inc}}(X)$ (a generalization of Abel's theorem). If $C^*_{\mathrm{hom}}(A \times X) = C^*_\tau(A \times X)$ for any Abelian variety, then $C^*_{\mathrm{Weil}}(X) = C^*_{\mathrm{abl}}(X)$ [487].

The factor group $C^r_{\mathrm{alg}}(X)/C^r_{\mathrm{Weil}}(X)$ is isomorphic with the Abelian subvariety $J^r_a(X)$ in $J^r(X)$ (which is called the r-th Picard variety in [536]). If $C^*_{\mathrm{hom}}(X) = C^*_\tau(X)$, then $J^r_a(X)$ parametrizes Poincaré's universal family of (n − r)-dimensional cycles and possesses the property of universality relative to analytic maps of the group $C^r_{\mathrm{alg}}(X)$ into Abelian varieties [536]. If $C^*_{\mathrm{hom}}(X \times J^r_a(X)) = C^*_{\mathrm{num}}(X \times J^r_a(X))$, then the Abelian varieties $J^r_a(X)$ and $J^{n-r-1}_a(X)$ are dual to each other.

The image $I_r(X)$ of group $C^r_{\mathrm{alg}}(X)$ relative to the map $\omega^r: C^r_{\mathrm{hom}}(X) \to T^r(X)$, where $T^r(X)$ is the r-th intermediate Griffiths Jacobian, is a subtorus of $T^r(X)$, possessing the structure of an Abelian variety [388]. $I_r(X)$ is contained in the maximal complex subtorus $A_r(X)$ of the image of the canonic homomorphism $H^{r-1,r}(X, \mathbb{C}) \to T^r(X)$. It is conjectured that $I_r(X) = A_r(X)$. A positive solution to this problem would allow us to prove the duality of the Picard-Griffiths varieties $\mathrm{Pic}^0_r(X)$ and $\mathrm{Pic}^0_{n-r-1}(X)$ (n = dim X). The last fact has been proved as yet only for r = (n + 1)/2 [388]. As Griffiths noted, $\omega^r(C^r_{\mathrm{hom}}(X))$, in general, is not contained in $A_r(X)$; it is precisely this observation that led him to construct the example of noncoincidence of $C^r_\tau(X)$ and $C^r_{\mathrm{hom}}(X)$.

2. Standard Conjectures. The question of the coincidence of the numerical and the homological equivalences is closely related with the so-called "standard conjectures" on alge-

braic cycles (see [400, 485]). The connection of the latter with the Weil conjectures on the ζ-function (respectively, with motif theory) is discussed in [485] (respectively, in [228]).

Later on $H^*(X)$ denotes the algebra of rational (if k = C) or l-adic rational cohomologies of a smooth n-dimensional projective variety over an algebraically closed field k. Let L_X denote a fixed polarization of variety X. The following two conjectures are called standard:

1) B(X): The correspondence $\Lambda_X \in H^{2n-2}(X\times X)$, associated with multiplication by the cohomology class L_X, is an algebraic cohomology class.

2) IP(X, L): The quadratic form $(a, b)\to L^{n-2p}a\cdot b$ determines the sign $(-1)^{n+1}$ on the subspace of algebraic primitive p-cycles [the cycle $x\in H^p_{\mathrm{alg}}(X)$ is called primitive if $L_X^{n-p}x = 0$].

We also consider the following statements.

3) C(X): The Kunneth components of the class of diagonals Δ in $H^n_{\mathrm{alg}}(X\times X)$ are algebraic.

4) A(X; L): The map $L_X^{n-2p}\cdot H^p_{\mathrm{alg}}(X)\to A^{n-p}(X)$ is bijective.

5) HodgeP(X): Let k = C, then $H^{p,p}(X, \mathbf{C})\cap H^{2p}(X)=H^p_{\mathrm{alg}}(X)$.

6) $D(X): C^*_{\mathrm{num}}(X)=C^*_{\mathrm{hom}}(X)$.

7) VL (the strong Lefschetz theorem): The map $L_X^{n-i}: H^i(X)\to H^{2n-i}(X)$ is an isomorphism for $0\leqslant i\leqslant n$.

It is known that B(X) is fulfilled for surface, flag varieties, complete intersections, and Abelian varieties [400, 535]. Conjecture IP(X, L) is fulfilled if k = C (Hodge's index theorem) or if $n\leqslant 2$ (Segre-Grothendieck; see [23]).

Conjecture $\mathrm{Hodge}^p(X)$ is always fulfilled, if p = 0, 1, n − 1, n (see [485]), for a cubic hypersurface in $\mathbf{P}^5$ and $\forall\ p$ (see [401]). In [401] Grothendieck shows that the generalized Hodge conjecture, connecting Hodge filtration on $H^i(X, Q)$ and "arithmetic" filtration (with respect to the codimension of the support of the cohomology class), is false in the general case. The connection of Hodge's conjecture with Tate's conjecture on algebraic cohomology classes is discussed in Deligne's report [209] (see survey [88] on Tate's conjecture). We remark that their equivalence for arbitrary Abelian varieties was recently proved by Pyatetskii-Shapiro [92] (Pohlmann [632] had proved this earlier for CM-type Abelian varieties).

Conjecture C(X) is true for Abelian varieties [535]. The strong Lefschetz theorem is valid if k = C (Hodge theory). As Deligne has proved recently (unpublished), the same is true in the general case too.

The following implications hold (see [485]): $B(X)\Rightarrow A(X, L)$; $D(X)\Rightarrow A(X, L)$; $A(X, L)\&I(X, L)\Rightarrow$ D(X); for all x and p, HodgeP(X) $\Rightarrow B(X)\&C(X)$ for all X and L, A(X, L) $\Rightarrow$ B(X); for all X, B(X) $\Rightarrow$ C(X).

From these implications it follows that for k = C conjecture D(X) is fulfilled for Abelian varieties and for any varieties with $n\leqslant 4$ [535]. In addition, the standard conjectures are equivalent to two other pairs of conjectures: $A(X)\&I(X, L)$ or $D(X)\&I(X, L)$.

3. Other Questions. The problem of the smoothing of algebraic cycles on smooth varieties is investigated in [422, 488]: For a given cycle Z find a cycle $Z' = \Sigma n_i Z_i$ such that $Z \underset{\mathrm{rat}}{\sim} Z'$ and the Z_i are smooth varieties. Hironaka [422] solves, modulo resolution of singularities (i.e., for example, if char k = 0), this problem positively for d-dimensional cycles, where $d\leqslant \min[3, (n-1)/2]$. Over a field of arbitrary characteristic he proves smoothing "with rational coefficients" ($\exists$ an integer m > 0 such that mZ is smoothed) for cycles with $d\leqslant (n-1)/2$. Kleiman [488, 493] proves an analogous result for $d\leqslant n/2$.

The connection between the cohomologies of variety X and its hyperflat section H is given by the classical "weak Lefschetz theorem": the canonic homomorphism $H^i(X, \mathbf{Q})\to H^i(H, \mathbf{Q})$ is bijective if $i\leqslant n-2$ and is injective for $i\leqslant n-1$. Its l-adic variant was proved in [654].

If H is a nonsingular hyperflat section of a smooth irreducible variety X of dimension $n\geqslant 3$, then Lefschetz' theorem on the hyperflat section asserts the bijectivity of the restriction homomorphism Pic X $\to$ Pic H (see survey [31]). In particular, Pic X $\simeq \mathbf{Z}$ for nonsingular complete intersection of X in P^n of dimension $n \geqslant 3$ and is generated by the class of hyperflat sections.

This theorem is obviously false for n = 2 (it is sufficient to consider the example of a quadric or a cubic). However, M. Noether proved that the latter assertion is preserved for a "general" surface of degree $d \geqslant 4$ in $\mathbf{P}^3$. In [214] Deligne proves the following stronger result. Let $X \subset \mathbf{P}^{2n+d}$ be a complete intersection of dimension 2n with $h^{2n,0}(X) \neq 0$. Then each algebraic class of l-adic cohomologies in $H_l^{2n}(X)$ is proportional to the class of H^n, where H is a hyperflat section. Several more general statements were proved earlier by Moishezon [78] in the case of k = C.

§4. Geometry of Families of Varieties

The method of stratifying an algebraic variety into subvarieties of lower dimension is a very important technical tool in the study of the geometry of varieties. On the one hand, it makes it possible to carry out induction with respect to dimension, and on the other, to extract information on specific properties of varieties from the properties of the critical points and degeneracies. In recent times it has with success been applied to solve old classical problems in algebraic geometry (by Griffiths when constructing the counterexample to the conjecture on the coincidence of algebraic and homological equivalence of cycles [380, 473]; by Deligne in proving Weil's conjecture, cf. [476]).

By an algebraic fibering or a family of algebraic varieties we shall mean, in what follows, a flat proper morphism $f: X \to S$ of regular schemes. Chapters 2 and 4 of the Grothendieck–Dieudonné "Eléments de géométrie algébrique" are devoted to the most general properties of such fiberings and a discussion of them can be found in survey [31]. Here we interest ourselves in the questions of mainly a global geometric nature, relating to certain special classes of families. We leave aside the applications to arithmetic of the questions being examined.

1. Monodromy of a Family. If a general fiber of an algebraic bundle $f: X \to S$ is smooth, then there exists a maximal open set $U \subset S$ such that $f \mid f^{-1}(U)$ is smooth, and consequently, $R^i f_* \mathbf{Q}_l \mid U$ is a locally constant sheaf. By the same token, for any point $s \in U$ there is defined the representation $\rho_l^i: \pi_1(U, \bar{s}) \to \mathrm{Aut}(H^i(X_{\bar{s}}, \mathbf{Q}_l))$, where $\pi_1(U, \bar{s})$ is a fundamental group (defined in the sense of Grothendieck if S is not a complex variety), while $X_{\bar{s}}$ is a geometric fiber of f over point s. If S is a smooth k-scheme, this representation is called a global monodromy of the family and is an important global invariant of the bundle. If S is an arrow with a closed point x_0 and a general point η, then $U = \eta$, $\pi_1(U, \bar{s}) = \mathrm{Gal}(k(\bar{\eta})/k(\eta))$ and ρ_l^i is called the local monodromy. In the complex case, instead of S we can consider the disk D, instead of x_0 its center 0, instead of U the punctured disk $D^* = D - \{0\}$, and instead of $\bar{\eta}$ any point $y \in D^*$; as a result we obtain the local monodromy $\rho^i: \pi_1(D^*, y) \simeq \mathbf{Z} \to \mathrm{Aut}(H^i(X_y, \mathbf{Q}))$. The image $1 \in \mathbf{Z}$ is called a monodromy transformation and its matrix T (defined to within conjugacy) is called the monodromy matrix. The monodromy theorem holds in this case: An N exists such that $(T^N - E)^{i+1} = 0$. The first (topological) proof of this theorem was given by Landman in 1966 (see [524]). Later a geometric proof was found by Clemens [204]. An algebraic version of this proof was independently given by Grothendieck ([700], Rept. I).

If $(\mathcal{H}^q_{DR}, \nabla)$ is a sheaf of relative de Rham cohomologies of a morphism f with a Gauss–Manin connection ∇ (see survey [31]), then the sheaf of horizontal sections of ∇ is $R^q f_* \mathbf{C}$. By the same token the monodromy is closely related with the corresponding connection ∇ (in the classical case this is simply the monodromy of the corresponding system of differential equations; see [208]). On the basis of this connection, Katz and Deligne gave an arithmetic proof of the monodromy theorem in the following stronger form: $\exists N > 0$, such that $(T^N - E)^r = 0$, where r is the number of pairs (p, q) with $p + q = i$ and $h^{p,q}(X_t) \neq 0$ (see [477]).

A study of the period map $\varnothing: D \to \Omega/\{T^k\}$ (see §6) and the methods of complex hyperbolic analysis allowed Borel to give an analytic proof of a theorem on the quasiunipotency of monodromies: The eigenvalues of T are roots of unity. This same statement was proved with the use of the theorem on the regularity of the Gauss–Manin connection in Deligne's book [208] (based on an idea of Brieskorn [12]). Recently Schmid [681] succeeded in considerably simplifying Borel's proof and in obtaining the monodromy theorem in the Katz—Deligne form. The theorem on the quasiunipotency of monodromies has a local analog which instead of X examines a sufficiently small neighborhood $\tilde{X}(x)$ of an isolated singular point x of a fiber X_0 of the morphism $f: X \to D$ and studies the monodromy transformation $\rho(x): \pi_1(D^*, y) \to \mathrm{Aut}(H^n(\tilde{X}(x)_y, \mathbf{Q})$, where $n = \dim X_0$. Its proof was found by Brieskorn [12, 179]. An algebraic analog of this theorem (the theorem on the quasiunipotency of an l-adic representation) was proved by Grothendieck ([101, 709]; also see [700], Rept. I). The representation $\rho(x)$ (as well as its global variant) is studied in the theory of the singularities of differentiable maps; a discussion of the results obtained would take us too far afield (see Arnol'd's survey [4]). A detailed study of a local monodromy on one-dimensional cohomologies of an Abelian variety was carried out by Grothendieck [399]. The theory of Néron's models plays an essential role here. In one special (but important) case the corresponding result was obtained by Igusa [448].

The local problem of invariant cycles (see [21]) is an interesting one. If $\rho^i: \pi_1(D^*, y) \to \mathrm{Aut}(H^i(X_y, \mathbf{Q})$ is a local monodromy, then the canonic homomorphism $H^i(X, \mathbf{Q}) \to H^i(X_y, \mathbf{Q})^{\pi_1}$ is surjective. It has been proved for a family of curves [21], surfaces (Katz, 1970, unpublished), while only a sketch of a proof is known in the general case, due to Deligne (unpublished).

If $\rho: \pi_1(U, s) \to \mathrm{Aut}(H^i(X_s, \mathbf{Q})$ is a global monodromy and $\bar{X}$ is a nonsingular compactification of X, then the canonic homomorphism $H^n(\bar{X}, \mathbf{Q}) \to H^i(X_s, \mathbf{Q})$ is surjective (Deligne [25, 210]). Hence it follows that $H^i(X_s, \mathbf{Q})^{\pi_1} = H^0(S, R^i f_* \mathbf{Q})$ is a Hodge sublattice (see §6) in $H^i(X_s, Q)$ not depending on the point $s \in U$. An analog of this statement for the case when S is a compact analytic variety was proved by Griffiths [383]. From this result he derived the following corollary: If section $\alpha \in H^0(U, R^n f_* \mathbf{C})$ is of type (p, q) at point s, then it is of type (p, q) everywhere. This statement, modulo the Hodge conjecture, proves Grothendieck's conjecture [395]: If α is an algebraic cohomology class at point s, then it is an algebraic cohomology class everywhere. A special case of this statement was proved in [166].

Another corollary is the following theorem of Grothendieck [398] and Borel—Narasimhan [174]: if $f: X \to S$ and $g: X' \to S$ are families of Abelian varieties and the morphism $u: R^1 f_* \mathbf{Z} \to R^1 g_* \mathbf{Z}$ is induced at some point $s \in S$ by the morphism $\tilde{u}_s: X_s \to X'_s$, of Abelian varieties, then u is induced by the unique morphism $\tilde{u}: X \to X'$ of the families.

Deligne [210] also proved the following important result: A global monodromy is a semisimple representation on any direct summand in $R^i f_* \mathbf{Q}$. In particular, hence it follows that the image $\pi_1(U, s)$ relative to the complex global monodromy $\rho: \pi_1(U, s) \to \mathrm{Aut}((R^n f_* \mathbf{C})_s)$ is a Lie group whose connected component is a semisimple group.

2. Theory of Vanishing Cycles. Seminars [700, 701] were devoted to an exposition of this theory. For any algebraic bundle $f: X \to S$ it studies the connection of the degeneracies of morphism f with the deviation of the sheaves of cohomologies $R^i f_* Q$ (or $R^i F_* Q_l$ in the general case) from the locally constant ones. In the simplest case, when the singularities of the fibers are degenerate quadratic points, this theory goes back to Picard and Lefschetz.

In what follows we assume that the base S is one-dimensional. If f is a smooth morphism, the specialization homomorphism $\mathrm{sp}^i: H^i(X_{\bar{s}}, \mathbf{Z}_l) \to H^i(X_{\bar{\eta}}, \mathbf{Z}_l)$ is bijective (here $\bar{\eta}$ is a geometric generic point, while $\bar{s}$ is a geometrically closed point of S). If S is a complex variety, then instead of l-adic cohomologies we can examine the usual integral cohomologies, while instead of $X_{\bar{\eta}}$ a "typical" fiber X_y of morphism f. In the general case Grothendieck constructs in [700] an exact sequence of Z_l-modules (respectively, Z-modules):

$$\ldots \to H^i(X_{\bar{s}}, \mathbf{Z}_l) \to H^i(X_{\bar{\eta}}, \mathbf{Z}_l) \to \Phi^i_{s,l} \to H^{i+1}(X_{\bar{s}}, \mathbf{Z}_l) \to \ldots$$

(respectively,

$$\ldots \to H^i(X_s, \mathbf{Z}) \to H^i(X_y, \mathbf{Z}) \to \Phi_s^i \to H^{i+1}(X_s, \mathbf{Z}) \to \ldots).$$

The module $\Phi_{s,l}^i$ (respectively, Φ_s^i) is a module of the vanishing l-adic (respectively, integral) cohomologies and measures the deviation of the sheaf $R^if_*\mathbf{Z}_l$ (respectively, $R^if_*\mathbf{Z}$) from the locally constant one in the neighborhood of point s. If $X_{\bar{s}}$ has only isolated singularities $x_1, \ldots, x_m$, then $\Phi_{s,l}^i = \oplus_j \Phi_{s,l}^i(x_j)$ and $\Phi_{s,l}^i(x_j) = 0$ for $i > n = \dim X_y$ and for $i < n$ (modulo resolution of singularities; for example, if char k(s) = 0 ([700], Rept. I)). This result is an algebraic analog of the more exact assertion of Milnor [77, 570]. Namely, by localizing f at point x_j, in the notation of Para. 1, the space $\tilde{X}(x_j)_y$ has the homotopy type of a bouquet (union) $\mu(x_j)$ of n-dimensional spheres (a local comparison theorem was proved in [213]: $H^i(\tilde{X}(x_j)_y, \mathbf{Z}) \otimes \mathbf{Z}_l \simeq \Phi_{s,l}^i(x_j)$). The number $\mu(x_j)$ (the Milnor number) is computed as the dimension of the C-space

$$O_{X,x_j}^{an} \Big/ \left(\frac{\partial f}{\partial t_1}, \ldots, \frac{\partial f}{\partial t_{n+1}}\right),$$

where f = 0 is the local equation of fiber X_s at point x_j, while $t_1, \ldots, t_{n+1}$ are the local parameters of a local analytic ring X at point x_j. In [212] Deligne proves an algebraic analog of this formula taking "wild branching" into account in char k = p > 0 (its hypothetical formulation had been independently found in [30]).

The sum $\sum_j \mu(x_j) = \chi(X_s) - \chi(X_{\bar{\eta}})$, where χ is the topological Euler characteristic, and

$$\chi(X) - \chi(X_{\bar{\eta}})\, \chi(S) = \sum_{s \in S} \sum_j \mu(x_j) = \sum_{s \in S} \chi(X_s) - \chi(X_{\bar{\eta}}).$$

A generalization of the first (respectively, second) equality to the case of not necessarily isolated points and char k > 0 was given in [461] (respectively, in [30]) (also see [212]).

An l-adic variant of the Picard–Lefschetz theory was presented in [215], relating to the study of vanishing cycles for the case when $S = \mathbf{P}^1$ and the fibers of morphism $f: X \to S$ have no more than one like double point (a Lefschetz sheaf). Each smooth projective variety has a birational model possessing such a sheaf [475]. If n = dim X, then $H^{n-1}(X, \mathbf{Q}_l) \hookrightarrow H^{n-1}(X_{\bar{\eta}}, \mathbf{Q}_l)$ (the weak Lefschetz theorem), and the orthogonal complement (relative to Poincaré duality) defines the subspace $E^{n-1}(X_{\bar{\eta}}) \subset H^{n-1}(X_{\bar{\eta}}, \mathbf{Q}_l)$ of the vanishing cycles. For any closed point $s \in S$ there exists an element $\delta_s \in E^{n-1}(X_{\bar{\eta}})$ (mapped, if n = 2m + 1, onto the generator of the group of vanishing cycles in Φ_s^{n-1}). All such δ_s generate $E^{n-1}(X_{\bar{\eta}})$. The action of the inertia group I at point s on $H^{n-1}(X_{\bar{\eta}}, \mathbf{Q}_l)$ is described by the Picard–Lefschetz formulas [215]. Deligne proved that a representation of a global monodromy acts on $E^{n-1}(X_{\bar{\eta}})$ absolutely irreducibly ([254] together with an unpublished proof of the strong Lefschetz theorem; see §3).

If n is even, then Poincaré duality induces a skew-symmetric bilinear form ψ on $H^{n-1}(X_s, \mathbf{Z})$, where X_s is a nonsingular fiber. Global monodromy preserves ψ and defines the symplectic representation $\rho: \pi_1(U, s) \to \mathrm{Sp}\,(H^{n-1}(X_s, \mathbf{Z}), \psi)$. The image ρ is Zariski-dense in $\mathrm{Sp}\,(H^{n-1}(X_s, \mathbf{Z}), \psi)$ [220, 746]. This fact plays an important role in Deligne's proof of Weil's conjecture on the ζ-function.

Applications of the Picard–Lefschetz theory to the theory of algebraic cycles (Griffiths' counterexample in case char k $\neq$ 0), to Noether's theorem (see §3), and to the theory of the ζ-function are given in [701].

3. Degenerate Fibers. A geometric classification of nonsmooth fibers of a family of algebraic varieties is known only in a few cases. Besides the one-parameter families of curves, classification is known for degenerate one-parameter families of two-dimensional Abelian varieties [735, 736] and for surfaces with a rational general fiber [75].

A complete classification, as far as families of curves are concerned, exists only for curves of genus $g \leqslant 2$. Here it is natural to restrict ourselves to the case of minimal models, i.e., exceptional curves of genus 1 are not contained in the fibers. The fundamental invariants of a degenerate fiber are: the geometric configuration of the irreducible components of the fiber, the corresponding intersection indices, the genera of the components, the multiplicities of their embedding in the fiber, as well as the local monodromy in a neighborhood of the fiber. In the case of curves of genus g = 0 there are no degeneracies. For g = 1 the degenerate fibers were described by Kodaira [43, 503] (k = C) and by Néron [608] (Kodaira's method was carried over to the arithmetic case by Deligne [218] and Néron [609]). In each case Kodaira computed the monodromy transformation. In case g = 2 and k = C a complete classification was given in [600-602]. The possible types were indicated (without existence proofs) earlier by Ogg [622]. As in the case of g = 1 the local monodromy transformation is an invariant of the degenerate fiber X_y. Restricting f to the disk $D \ni y$, we obtain the map $s\colon D^* \to M_2$ into the moduli variety M_2 of curves of genus 2, continued upto the map $\bar{s}\colon D \to \bar{M}_2$, where $\bar{M}_2$ is a compactification of M_2. In [600] it was shown that f is determined locally by its own monodromy, by the image $\bar{s}(0) \in \bar{M}_2$ and by the order of the function $\bar{s}^*(\varphi)$ at 0, where $\varphi = 0$ is the local equation of $\bar{M}_2 \setminus M_2$ at point $\bar{s}(0)$.

A construction of the compactification $\bar{M}_g$ of the moduli variety M_g of curves of genus g, based on the concept of a family of stable curves, was given in [222] (also see [651] for an account of this paper). Such is the name given to families of curves (over not necessarily a one-dimensional basis), whose degenerate fibers have only double like points, while each irreducible rational component intersects the other components at no less than three points. A rough moduli space $\bar{M}_g$ exists, parametrizing stable curves of arithmetic genus g over field k. The moduli scheme $\mathfrak{M}_g$ of smooth curves is contained in $\bar{M}_g$ as an open set; $\bar{M}_g$ is a normal and proper algebraic space [651] over Z.

An important role in the proof of the properness is played by the theorem on stable reduction of curves: after a certain ramified covering of the base the minimal model of a general fiber is a family of stable curves. The proof of this theorem was given in [222] (using the analogous theorem of Grothendieck for Abelian varieties [399]), and also by Artin and Winters (see [144, 651]). The latter proof contains interesting general results on the structure of degenerate fibers for an arbitrary genus $g > 0$.

4. Families of Curves. The theory of one-parameter families of curves is closely connected with the theory of surfaces and the theory of algebraic curves over the field of algebraic functions of one variable. An important role is played here by the concept of a minimal model of a bundle (see Para. 3). The arithmetic applications of this theory are connected first of all with Mordell's theorem and with finiteness theorems (see Parshin's surveys [88, 629]). We remark on the recent significant progress in this area achieved by Arakelov [3]. By developing Parshin's techniques [87] (and, in particular, using the theory of families of stable curves; see Para. 3), he succeeded in proving Shafarevich's conjecture: there exists only a finite number of nonisomorphic nonconstant bundles of curves of genus $g > 1$ with a fixed set of degeneracies on the base (a complete algebraic curve over an algebraically closed field of characteristic zero). This assertion was proved in [87] for bundles with an empty set of degeneracies.

Besides arithmetic application, [3, 87, 89] contain interesting geometric facts on fiberings into curves over a field k of characteristic 0. For example, the following isotriviality criterion is proved in [87]: A smooth bundle $f\colon X \to B$ of curves of genus $g > 1$ over a complete curve B is isotrivial if and only if the degree of bundle $R^1 f_* O_X$ equals zero. This is true even in case $\operatorname{char} k > 0$ (Parshin, unpublished). If the genus of curve B equals 0 or 1, then this condition is always fulfilled in $\operatorname{char} k = 0$. Examples of smooth nonisotrivial bundles have been presented by Kodaira (see [87]).

If $f\colon X \to B$ is a bundle with a general fiber of genus g, then sheaf $R^1 f_* O_X$ is not always locally free. However, this is so if f is cohomologically flat (i.e., $f_* O_X = O_B$ is universal for any change of base). The latter condition is fulfilled if the greatest common divisor of the multiplicities of the degenerate fibers are relatively prime with the characteristic of the ground field k [650, 652]. If f is cohomologically flat, then the Riemann–Roch–Grothendieck theorem yields the equality

$$\chi(X, O_X)=(1-g)\chi(B, O_B)-\deg(R^1 f_* O_X).$$

In particular, the Euler characteristic is multiplicative for isotrivial bundles. The ampleness of the sheaf $\Omega^1{}_X/_B$ on X if f: X→B is a smooth nonisotrivial bundle of curves of genus g > 1 over curve B of genus $q\geqslant 2$ is proved in [87]. This fact was generalized by Arakelov to families of stable curves [3].

Little is known about multiparameter fiberings into curves. The only general result is an assertion on the purity of the set of degeneracies (the projection of the set of points of nonsmoothness of the morphism forms a divisor on the base). Its proof is given in [29, 231] and in [647, 720] if the ground field is the complex number field. Families of rational curves over a two-dimensional base were investigated in [143] and of elliptic ones in [478].

By generalizing Tate's theory of p-adic uniformization of elliptic curves, Mumford constructed a beautiful theory of uniformization of curves over a local base [57, 590]. This is a certain analog of the classical Schotky uniformization. If A is a complete local ring with quotient field K, then the group PGL(2, K) acts birationally on $\mathbf{P}^1_A$. Roughly speaking, in order for it to act "well" it is necessary to blow-up all the points of indeterminacy of all maps $\gamma\in PGL(2, K)$ onto a closed fiber. By selecting the subgroups $\Gamma\subset PGL(2, K)$ in a specified manner and by factoring the action with respect to them, we can obtain curves of any genus X→Spec A, whose closed fibers are strongly degenerate, i.e., consist of a finite number of rational curves intersecting transversally and having only like double singularities.

§5. Moduli of Algebraic Varieties

The problem of moduli of algebraic varieties has been the subject of numerous papers in the classical literature. However, until recently only the analytic theory of moduli of curves (Teichmuller's theory) and of Abelian varieties (the upper Siegel semispace with a modular group acting on it and the theory of the ζ-function) were the most satisfying. The algebro-geometric theory was especially birational and reduced mainly to counting the number of complex parameters (moduli) on which a general variety of given type depends. Riemann already had established that the number of moduli of curves of genus g > 1 equals 3g–3. The very concept of a "moduli variety" had not even been defined in the problem of the existence of a moduli variety (for example, see [580]). A precise statement and understanding of the problem became possible thanks to the original ideas of Kodaira and Spencer [511–513] and to the general theory of deformations of structures and of Grothendieck's representable functors (see survey [31]). Mumford did this in a series of remarkable papers [578–586]. The basic definitions and original results are presented in his book [580]. The problem proved to be so difficult that the existence of even a "crude" (see Para. 3 below) moduli scheme has been proved as yet only for curves, Abelian varieties, and certain special varieties. As Mumford writes (see the Appendix to Chapter V in [757]), the state-of-the-art is such that in many cases the moduli scheme, in general, does not exist, and where it could exist it still has not been proved that it does exist. In separate cases its existence is known as a complex-analytic space or as an Artinian algebraic space (see [651, 637]).

1. Statement of the Problem and Some General Results. The most essential feature in the contemporary treatment of moduli theory is its separation into a local and a global theory, as well as the introduction of the important concept of a polarized variety (Matsusaka [556], Mumford [580]). Only polarized varieties vary well in algebraic families. The construction of a moduli theory of unpolarized varieties (curves, for example) is possible in the majority of cases, only because some canonic polarization exists on them.

Let X_0 be a complete smooth variety over an algebraically closed field k. The pair $(X_0, \mathfrak{P}_0)$, where $\mathfrak{P}_0\in \mathrm{Pic}\, X_0/\mathrm{Pic}^0 X_0$ is the class of a certain ample invertible sheaf $L\in\mathrm{Pic}\, X_0$, and $\mathrm{Pic}^0 X_0$ is the connected component of the Abelian Pic X_0, is called a polarized variety.

A smooth projective morphism of schemes f: $X\to S$, where S is a Noetherian scheme, while the fibers are algebraic varieties, is called a family of varieties with base S. We sometimes consider not necessarily smooth families (for example, flat ones, etc.) (see [222, 104]).

The pair $(X/S, \mathfrak{P}/S)$ is called a polarized family, where X/S is the family f: $X\to S$ with base S, while $\mathfrak{P}/S$ is the class of a relatively ample invertible sheaf $L_{X/S}$ in Hom(S, Pic X/S) modulo Hom(S, $\mathrm{Pic}^0 X/S$), where Pic X/S is a relative Picard scheme (see survey [31]).

The main idea in the construction of a global moduli theory consists in the following. Let there be given a collection $\mathfrak{M}$ of objects with fixed discrete invariant (for example, curves of genus g with canonic polarization, polarized Abelian varieties of dimension n and degree d, n-dimensional polarized varieties with a given Hilbert polynomial, etc.) whose moduli have to be constructed. For this we examine all possible polarized familes $(X/S,\mathfrak{P}/S)$ whose geometric fibers with the induced polarization belong to $\mathfrak{M}$ and we construct the functor

$$\mathcal{M}(S)=\left\{\begin{array}{l}\text{set of polarized families } (X/S,\ \mathfrak{P}/S)\\ \text{to within isomorphism over } S\end{array}\right. .$$

With each morphism h: $T\to S$ we associate a map $h^*:\mathcal{M}(S)\to\mathcal{M}(T)$ by taking an induced family.

Let M be an object in the category of algebraic varieties, schemes, algebraic spaces, analytic spaces (in a theory over C), etc. and let h_M be a functor of points in the corresponding category: $h_M(S) = \mathrm{Hom}(S, M)$.

1. If functor $\mathcal{M}(S)$ is representable, i.e., $\mathcal{M} = h_M$ for some M, then in this case a universal family with base M exists and M is called a finite object in moduli theory.

2. If a morphism $\varphi:\mathcal{M}\to h_M$, exists for some M, possessing the two properties:

a) For each algebraically closed field k the morphism φ induces the isomorphism $\varphi(\mathrm{Spec}\, k):\mathcal{M}(\mathrm{Spec}\, k)\cong h_M(\mathrm{Spec}\, k)$;

b) for each object N and morphism $\psi:\mathcal{M}\to h_N$ there exists a unique morphism $\chi:h_M\to h_N$ such that $\psi=\chi\circ\varphi$, then object M is called a "crude" (global) moduli space (respectively, variety, scheme, etc.).

In this case a universal family does not, in general, exist on M, but the set of geometric points of M is found to be in a natural one-to-one correspondence with the set of all polarized varieties from $\mathfrak{M}$ (to within isomorphism), defined over algebraically closed fields [580].

The functor $\mathcal{M}$ is representable in very few cases (for example, for curves of genus $g>1$ without automorphism). It is not representable even in the case of all smooth curves of genus $g\geqslant 1$ (see [582]). However, if we examine the functor of polarized families of curves (or of Abelian varieties) with an additional structure, namely, "rigidity," then it proves to be representable (see below).

The standard plan for constructing a crude moduli space reduces to the sequential solving of the following two problems.

A. The construction of a sufficiently large (complete) polarized family (containing to within isomorphism all the objects from $\mathfrak{M}$) f: $X\to H$ as a universal family (of deformations) of the subvarieties of the projective space P^n, i.e., the construction of H as some subscheme of the Hilbert or Chow scheme (see survey [31] on the latter).

B. The construction of the factor M = H/R by the equivalence relation R identifying the point from H, the fibers over which are isomorphic.

In the case of polarized families with rigidity the family f: $X\to H$ of problem A can be given also over some covering of a subscheme of the Hilbert scheme or over a subscheme with product of the Hilbert scheme (see [580]).

One of the first questions arising in the attempt to solve problem A is that of the simultaneous immersion of polarized varieties with a fixed numerical invariant into a projective space. More precisely, let $(X_0, \mathfrak{P}_0)$ be a polarized variety, $L_0\in\mathfrak{P}_0$ be an ample invertible sheaf, and $h(n)=\chi(X_0, L_0^n)$ be a Hilbert polynomial. Then if $(X/S, \mathfrak{P}/S)$ is a polarized family with a connected base S and relatively ample invertible sheaf $L_{X/S}$, containing (X_0, L_0), then the Hilbert polynomial $h_s(n)=\chi(X/S, L_{X/S}^n)$ does not depend on $s \in S$ and coincides with h(n) (see [580]). Does a constant C, depending only on the Hilbert polynomial h(n), exist such that when $n > C$ the sheaves L_s^n are very ample for all polarized varieties $(X_s, \mathfrak{P}_s)$ with polynomial h(n) and $H^i(X_s, L_s^n)=0$ with $i > 0$? An affirmative answer to this question has been known for a long time for curves and Abelian varieties (see [49, 50]). In the case of curves C depends only on the genus g of the curve. For Abelian varieties C = 3. One of the fundamental theorems of Matsusaka and Mumford [560] asserts the existence of such a constant for polarized surfaces. Matsusaka [555] proved this in the case of arbitrary smooth polarized varieties over an algebraically closed field of characteristic 0. He had previously [558, 559] proved the existence of such a constant for canonically polarized varieties (i.e., with an ample canonic sheaf) over a field of characteristic 0. The last result was obtained by Tankeev [103] also.

For a surface of fundamental type with a canonic polarization (not necessarily smooth; admitting of isolated rational singular points) in characteristic 0 the constant C does not depend even upon the Hilbert polynomial, i.e., is an absolute constant. This follows from the results of Kodaira [500] and Bombieri [170, 171] (also see [104]).

Another question in problem A is the following: Is the type of the variety preserved under deformations (in smooth polarized families, for example)? Grothendieck (also see [580]) showed that a smooth algebraic deformation of an Abelian variety is an Abelian variety. An analogous result for surfaces of type K3 can be found, for example, in [637]. See [377] on deformations of a projective space.

The scheme H in problem A can be reducible, unconnected, singular, and even unreduced (see Para. 2 below). Even in concrete cases of surfaces of fundamental type we do not know which invariants separate the irreducible components of scheme H.

In problem B the equivalence relation on H is usually given by the action of an algebraic group G [most often PGL(n)]. The problem of the existence of the factor is a very difficult one. It is discussed in detail in Mumford's book [580] (also see survey [31]). Mumford introduces the concepts of a category functor and a geometric one. The construction of a crude moduli space is reduced to the most difficult and highly subtle problem of the existence of the geometric factor. See Para. 3 below for those few cases in which it is known to exist. The concept of stability, introduced by Mumford, corresponding to the concept of orbits of general position, is a very valuable one for moduli problems.

The first fundamental theorem of Matsusaka and Mumford [560] states that the isomorphism of complete smooth polarized varieties (not ruled) over a discrete valuation ring, given over a general point of this ring, can be continued upto isomorphism over the whole ring. This fact is used to verify the closedness and properness of the action of group G on H.

The nonexistence of moduli varieties even in good cases compelled Matsusaka to introduce the concept of a Q-variety, generalizing the usual variety as much as necessary for the moduli problem. As a Q-variety the crude moduli variety exists in all cases when there exists the "universal family of algebraic deformations" (in the sense of Matsusaka), i.e., when problem A is solvable.

Besides algebraic curves and Abelian varieties (see Para. 3) the solution of the crude moduli problem is known in the following cases:

1) exists as a complex space: for polarized surfaces of type K3 (see [96]) — in this case M is a 19-dimensional complex variety; for surfaces of fundamental type (see [104]) — here the existence of only M_{red} has been proved, i.e., only of the reduced subspace. It is not known whether M coincides with M_{red}. An example when M is reducible exists;

2) exists as an algebraic space over C: in [637] Popp proved the existence of the crude moduli space for polarized surfaces K3 and for canonically polarized varieties with a very ample canonic sheaf — in the last case the existence of only M_{red} was proved.

2. Local Theory and Deformations. Local moduli theory studies "infinitesimal" deformations of a fixed variety X_0 [or of a polarized variety $(X_0, \mathfrak{P}_0)$] and its main problem is the

construction of a universal family of such deformations. Kodaira and Spencer [511-513] examined it from the analytic point of view. Their fundamental theorem, improved later by Kuranishi [516] and Douady [233], is the following: There exists a proper smooth morphism of complex-analytic spaces

$$u: X \to S$$

with a fixed point $s_0 \in S$, an isomorphism $u^{-1}(s_0) \simeq X_0$ and a local base S (i.e., instead of S we can examine any small neighborhood $S' \ni s_0$ in S), such that:

a) any other small deformation is induced by deformation u: $X \to S$;

b) the Kodaira–Spencer map φ: $T_{s_0, S} \to H^1(X_0, T_{X_0})$ is an isomorphism, where T_{X_0} is the sheaf of germs of sections of a tangent bundle, while $T_{s_0,S}$ is the Zariski tangent space to S at point s_0.

In other words, there always exists a versal family of infinitesimal deformations. If in condition a) each deformation $X' \to S'$ is induced by the one morphism $S' \to S$, then $X \to S$ is a universal family. This is so if $H^0(X_0, T_{X_0}) = 0$.

We note that the base S of a versal family can be singular, reducible, and even unreduced. Locally it is given in the space $H^1(X_0, T_{X_0})$ by no more than dim $H^2(X_0, T_{X_0})$ equations (Kuranishi [516]), i.e., dim $H^2(X_0, T_{X_0})$is the largest number of obstructions to the continuation of first-order infinitesimal deformations to the present deformations in a neighborhood of point s_0. If $H^2(X_0, T_{X_0}) = 0$, there are no obstructions and S is nonsingular at s_0 and is reduced.

An analog of this theorem in formal geometry, suitable for all characteristics and for any scheme, was worked out by Grothendieck and Schlessinger (see survey [31])..

Let us now assume that a smooth complete variety X_0 over C is rigged with a polarization $\mathfrak{P}_0$. Then in the base S of a versal family $X \to S$ there exists the largest closed subspace $S_{\mathfrak{P}} \subset S$, over which the polarization $\mathfrak{P}_0$ is lifted onto $X_{\mathfrak{P}} = u^{-1}(S_{\mathfrak{P}})$. The family $u_{\mathfrak{P}}: X_{\mathfrak{P}} \to S_{\mathfrak{P}}$ is then a versal family of local deformations of the polarized variety $(X_0, \mathfrak{P}_0)$. If a canonic polarization exists on X_0, then it can be lifted onto the whole family $X \to S$. In the general case there can be points $s \in S$, not falling into any one of the closed subspaces $S_{\mathfrak{P}}$ for all possible polarization $\mathfrak{P}_0$ on X_0 and the fibers $u^{-1}(s)$ over them are not algebraic varieties. In formal geometry this corresponds to the nonalgebraizability of the formal scheme of versal form deformations [6]. The simplest examples where algebraic deformation does not exist are Abelian varieties and surfaces of type K3 (e.g., see [2], Chap. IX).

Thus, from the Kodaira–Spencer–Kuranishi theory (and its formal analog) it follows that the construction of a local moduli space reduces to an analysis of the obstructions:

a) to the continuation of infinitesimal deformations of order k (k = 1, 2,...) onto higher orders: the obstructions lie in $H^2(X_0, T_{X_0})$;

b) to lifting of the polarization from X_0 onto $X \to S$: the obstructions lie in $H^2(X_0, O_{X_0})$ (e.g., see [2], Chap. IX, and [757], Appendix to Chap. V).

For a smooth curve X_0 of genus $g \geqslant 2$

$$\dim H^0(X_0, T_{X_0}) = \dim H^2(X_0, T_{X_0}) = 0,$$
$$\dim H^2(X_0, O_{X_0}) = 0, \quad \dim H^1(X_0, T_{X_0}) = 3g-3,$$

therefore, there exists a universal local nonsingular moduli variety of dimension 3g–3.

In an Appendix to Chapter V of book [757] Mumford gives a simple explanation from the contemporary position of the classical results relating to the estimation of the number of moduli for algebraic surfaces (also see Kodaira [502, 506]. Simplicity here, as Mumford explains, is connected with the fact that Italian geometers restricted their consideration to deformations of first order only. The greater difficulties arise when studying the obstructions — this is the classical problem of the "completeness of the characteristic system" (see [8, 49, 757]).

A few but very interesting examples show that obstructions arise when constructing the local moduli space already for surfaces over C. Mumford [757] points out that such an example can be obtained by combining his results [589] with a result of Kodaira [502]. Katz [471] constructed a class of elliptic surfaces with obstructions. In [472] he, conversely, shows that Kodaira surfaces [508] (irregular surfaces possessing a smooth morphism on a complete curve) do not have obstructions under deformations.

In a recent very interesting paper Horikawa (RZh Mat, 1974, 5A626) studies the deformations of simply connected algebraic projective surfaces over C with $p_g = 4$ and $(K\cdot K) = 5$ (K is the canonic class). Two types of such surfaces are indicated: surfaces of degree 5 in P_3 and certain two-sheeted coverings of a rational ruled surface. They yield two transversally intersecting irreducible components (globally) of dimension 40. The Kuranishi space at the points of their intersection is of dimension 41 — the corresponding obstructions are not computed explicitly. Horikawa's paper is more remarkable in that it gives an example of a reducible moduli variety for surfaces with fixed numerical invariants (and even simply connected). The moduli scheme is always irreducible for curves [26].

In an appendix to book [757] Mumford derives the following estimate for the number ω of obstructions to deformations of polarized varieties

$$\omega \leqslant \dim H^2(X_0, E_{\mathfrak{P}_0}),$$

where $E_{\mathfrak{P}_0}$ is a locally free sheaf defined from the extensions

$$0 \to O_{X_0} \to E_{\mathfrak{P}_0} \to T_{X_0} \to 0,$$

of an appropriate class of polarizations $\mathfrak{P}_0$ in $H^1(X_0, \Omega^1_{X_0}) \simeq \mathrm{Ext}(T_{X_0}, O_{X_0})$.

Local theory is employed in global moduli theory already at the level of problem A (see Para. 1) for the verification of the completeness of the family $X \to H$ and the smoothness of base H. A variant of the Kodaira—Spencer—Grothendieck deformation theory for a nonsingular subvariety X_0 inside a complete nonsingular variety Z is found as follows. Let

$$X \subset Z \times S$$

be any flat family over S of subvarieties in Z, containing X_0 as a fiber over a point $s_0 \in S$. Then there is defined the characteristic map

$$\varphi' : T_{s_0} \to H^0(X_0, N),$$

where T_{s_0} is the Zariski tangent space to S at point s_0, while N is a normal sheaf of X_0 in Z (see Kodaira [506], Mumford [49, 581]). If φ' is surjective, then the family $X \to S$ is said to be complete; if $H^1(X_0, N) = 0$, then S is nonsingular at point s_0 (see [581]). For complete families $H^0(X_0, N)$ can be interpreted as first-order infinitesimal deformations of X_0 in Z and the obstructions to their continuation lie in $H^1(X_0, N)$.

When X_0 is a curve on a smooth projective F over an algebraically closed field of characteristic 0, obstructions exist only for "flat" curves. In characteristic $p > 0$ the existence of obstructions is connected with the fact that the scheme Pic F can be unreduced. These questions were studied in full in Mumford's book [49], i.e., the problem of the "completeness of the characteristic series" has been completely solved for curves and surfaces.

In [589] Mumford shows that the Hilbert schemes of smooth curves of genus 14 and degree 24 in P^3 over C is not reduced: Nilpotent elements in the structure sheaf exist even on some open subset.

Let $X \to H$ be a family of polarized variety, being a solution of problem A, and let $H \subset \mathrm{Hilb}\, P^n$ be a subscheme of the Hilbert scheme. Then, for any geometric point $h \in H$ and fiber X_h we have the maps

$$\begin{tikzcd} & H^0(X_h, N_{P^n/X_h}) \arrow[d, "d"] \\ T_{h,H} \arrow[ur, "\rho'"] \arrow[r, "\rho"'] & H^1(X_h, T_{X_h}), \end{tikzcd}$$

where map d is induced by the exact sequence of sheaves

$$0 \to T_{X_h} \to T_{\mathrm{P}^n/X_h} \to N_{\mathrm{P}^n/X_h} \to 0,$$

carrying the most essential information on the local structure of the moduli variety.

In [96] it was shown that the variety H is nonsingular for polarized surfaces of type K3. It is nonsingular also for stable curves of genus $g \geqslant 2$, embedded in P^n with the aid of a 3-fold sheaf (see [26, 222]).

Small deformations of complete intersections were studied in [378, 556].

3. Moduli of Curves and Abelian Varieties. The greater part of the new results here are due to Mumford [578-588]. A brief survey of them was given in his report at the International Congress of Mathematicians in Nice [586].

In [580] Mumford showed that for smooth curves of genus $g \geqslant 0$ there exists a crude moduli M_g over **Z**, being quasiprojective, normal, and reduced over each open subset Spec Z − (p) in Spec **Z**, p is a prime.

Since a finite object does not exist for the moduli of curves of genus $g \geqslant 1$, a certain replacement for it — an algebraic stack — was introduced in [582, 222]. This is a category of schemes with Grothendieck's étale topology and with certain additional conditions. The agebraic stack of the moduli of curves possesses good universal properties and with its aid we can obtain geometric information on the crude moduli scheme of curves. In [26, 222] Deligne and Mumford study the stack of stable curves of genus $g \geqslant 2$ (the consideration of stable, and not merely smooth, curves allows us to obtain a compactification of scheme M_g). It is proved that the moduli scheme M_g of smooth curves is geometrically irreducible. The irreducibility of $M_g \otimes \mathbf{C}$ was known previously (for example, from Teichmuller's theory). In characteristic $p > 2g+1$ the geometric irreducibility of $M_g \otimes \mathbf{Z}/p\mathbf{Z}$ was proved by Fulton [256] as well with the aid of constructing the Hurwitz scheme of coverings of curves over P^1 with specified branching points.

Raynaud [651] obtained one more proof of the irreducibility of M_g, having shown that the crude moduli space $\overline{M}_g$ of stable curves exists in the category of algebraic spaces. Finally, in [586] announced that he, Knudsen, and Seshadri had proved that the moduli space $\overline{M}_g$ of stable curves of genus $g \geqslant 2$ is a projective scheme over Z. The moduli scheme M_g of nonsingular curves is contained in $\overline{M}_g$ as an open subscheme.

Rauch has shown that the singular points of the variety $M_g \otimes \mathbf{C}$ for $g > 3$ corresponds to those and only those curves of genus g over C, which have nontrivial automorphisms; for $g = 3$ we need to exclude further those hyperelliptic curves which have only two automorphisms. In characteristic $p > 0$ these results were extended by Popp [634] (the case of $g = 3$ has been investigated in [634] but not upto completion). The lattice M_g of singular points was studied in detail by Igusa [449] for $g = 2$.

The classical problems of the universality and the rationality of the moduli variety $M_g \otimes \mathbf{C}$ remain open. The universality has been proved for $g < 11$ by Severi (see the reference in [757]; also see [111]). Tyurin (unpublished) also proved this fact. The rationality of M_g is not known for any $g \geqslant 2$ whatsoever.

In [587] Mumford showed that the Albanese variety of variety $M_g \otimes \mathbf{C}$ is trivial, i.e., is just the same as for rational varieties. He showed that $\Gamma/[\Gamma, \Gamma]$ is a finite cyclic group whose order divides into 10, where Γ is Teichmuller's modular group acting on the Teichmuller space of curves of genus g, while $[\Gamma, \Gamma]$ is its commutant. This is equivalent to the Picard group of the algebraic stack $\mathcal{M}_g$, classifying curves of genus $g \geqslant 2$, being isomorphic to $H^2(\Gamma, \mathbf{Z})$. The equalities $\mathrm{Pic}(\mathcal{M}) = \mathbf{Z}/10\mathbf{Z}$, for $g = 2$ and $\mathrm{Pic}(\mathcal{M}) = \mathbf{Z}/12\mathbf{Z}$ for $g = 1$ have been proved (see [51, 582]).

Let $f\colon X\to S$ be a smooth family of curves of genus $g\geq 2$. Let an integer $n>0$ be invertible on S; then the sheaf $R^1f_*(\mathbf{Z}/n\mathbf{Z})$ is locally free in the étale topology of scheme S and has rank 2g. Multiplication in cohomologies provides $R^1f_*(\mathbf{Z}/n\mathbf{Z})$ with a locally nondegenerate symplectic form with values in $\mathbf{Z}/n\mathbf{Z}$, defined to within multiplication by invertible elements from $(\mathbf{Z}/n\mathbf{Z})^*$.

The Jacobian structure of level n on X is the giving of the symplectic isomorphism (to within a factor)

$$R^1f_*(\mathbf{Z}/n\mathbf{Z})\rightleftarrows(\mathbf{Z}/n\mathbf{Z})^{2g}.$$

Let $\mathcal{M}_{g,n}$ be a functor of smooth families of curves with a Jacobian structure of level n. It turns out that when $n\geqslant 3$ this functor is representable by a quasiprojective scheme $M_{g,n}$ over $\operatorname{Spec}Z[1/n]$ (see [580]) and there exists a finite Galois covering $M_{g,n}\to M_g$ over $\operatorname{Spec}Z[1/n]$. Scheme $M_{g,n}$ is smooth for sufficiently large n (Popp [635]).

The connection of the moduli of curves with the moduli of Abelian varieties was established by Torelli's theorem: A curve is uniquely determined by its own Jacobian variety J with Poincaré polarization θ.

Mumford laid down the origin of the general algebraic theory of the moduli of Abelian varieties in [580]. Let X_0 be an Abelian variety of dimension g over an algebraically closed field k and L_0 be an ample invertible sheaf on X_0. Then there is defined the homomorphism $\Lambda(L_0)\colon X_0\to\hat{X}_0=\mathrm{Pic}^0X_0$, depending only on the class L_0 in $\mathrm{Pic}\,X_0/\mathrm{Pic}^0X_0$. Its nucleus is a finite group of order d^2, where d is some integer (if $\deg L_0$ is prime with char k). The number d^2 is called the degree of the polarization specified by $\Lambda(L_0)$.

Now let $X\to S$ be an Abelian scheme of dimension g over the scheme S and the null section $\varepsilon\colon S\to X$. Mumford calls the homomorphism $\lambda\colon X\to\hat{X}$, which has the form of Λ on each geometric fiber, a polarization on X. There does not always exist a relatively ample invertible sheaf L such that $\lambda=\Lambda(L)$; in this the definition of polarization for Abelian schemes differs from the common definition for families of algebraic varieties (see Para. 1). However, some relatively ample invertible sheaf $L(\lambda)$ can be canonically associated with any polarization λ.

For each integer $n\geqslant 1$, different from the characteristic of the residue field of scheme S, the structure of level n on X/S is the giving of 2g section $\sigma_1,\ldots,\sigma_{2g}$ of an Abelian scheme $X\to S$ such that:

1) For all geometric points $s\in S$ the images $\sigma_i(s)$ comprise a (canonic sympletic) base of the group of points of order n on fiber X_s;

2) $\psi_n\circ\sigma_i=\varepsilon$, where $\psi_n\colon X\to X$ is a morphism of multiplication by n.

Let $\mathcal{A}_{g,d,n}(S)$ be a functor and $A_{g,d,n}$ be a crude moduli space of Abelian varieties with a polarization λ of degree d^2 and structure of level n. Mumford [580] showed that $A_{g,d,n}$ is for all (g, d, n) a quasiprojective scheme over an open set of the form Spec Z − (p). For $n\geqslant 3$ it is even a finite moduli scheme.

More profound study of the moduli of Abelian varieties was continued by Mumford [55-57, 584-586]. The main purpose of these papers was to work out a purely algebraic analog of the analytic theory of theta functions. The idea is to study the functors of families of Abelian varieties with a finer "non-Abelian" structure, a θ-structure of finite level δ, and to pass to the limit relative to isogenies. As a result there is obtained a simultaneous embedding of all Abelian varieties with a structure of type δ into a projective space, while the null section of this maximal family is a moduli variety which is given as an open subset, given explicitly with the aid of the Riemann theta-relations, of some projective variety. Furthermore, when passing to the limit the boundary of the corresponding open moduli subscheme is given by explicit equations. The θ-functions arising here are defined on Tate's 2-adic space of the Abelian variety X, while the coordinates of the canonic immersion are interpreted as theta constants. Thus, there emerges a purely algebraic and very meaningful theory, well consistent with the analytic theory of uniformization of Abelian varieties and their moduli spaces. The analytic aspect of the theory was developed earlier by Baily [150] and Igusa [445, 451, 452].

The next stage in the moduli theory of curves and of Abelian varieties consists in the study of the structure of the boundaries of moduli varieties, and also in a clear description of the moduli subvarieties of the Jacobians of curves in all the moduli varieties of Abelian varieties. A comparison of the curve of its Jacobian with the Poincaré polarization defines the morphism of functors

$$j:\mathcal{M}_g\to\mathcal{A}_{g,1,1}$$

and

$$j_n:\mathcal{M}_{g,n}\to\mathcal{A}_{g,1,n}$$

for any level n. By Torelli's theorem they are injective for all geometric points and, consequently, yield the embeddings

$$j:M_g\hookrightarrow A_{g,1,1},$$
$$j_n:M_{g,n}\hookrightarrow A_{g,1,n}.$$

The map $j: M_g\to A_{g,1,1}$ is not closed; as Hoyt [437] showed, to the points from its closures correspond g-dimensional products of Jacobians. More precisely (see [54, 586]), let $\tilde{M}_g\subset\overline{M}_g$ be a subset in the moduli variety of stable curves, corresponding to a stable curve consisting of nonsingular components and moreover, let their graph be a tree. Then j can be continued upto a proper map $\tilde{j}:\tilde{M}_g\to A_{g,1,1}$. The classical problem consists in how to describe the image $\tilde{j}(\tilde{M}_g)$ in $A_{g,1,1}$ by explicit equations. The paper [250] by Farkas and Rauch, as well as the very interesting papers of Andreotti and Mayer [136] and Mayer [567], were devoted to it.

In the case of g = 4 the image of $\tilde{M}_4$ in $A_{4,1,1}$ is given by one equation. Schotky (as far back as 1888) found the euqation of the theta-constant whose null set contains M_4. Mumford [54] explains that as a matter of fact Schotky's equation describes the same. Andreotti and Mayer found analytic equations giving the subsets V_g^r in $A_{g,1,1}\otimes\mathbf{C}$, for which the corresponding Abelian varieties possess the property: The singularities of their polarization divisor have codimension $\leqslant$ r. It is proved that $\tilde{M}_g\otimes\mathbf{C}$ is contained in V_g^3. The reason for the arising here of equations of the wave type is explained in [567].

Over the complex number field $A_{g,1,1}\otimes\mathbf{C}$ is represented as a factor of the upper Siegel halfspace, i.e., all complex symmetric matrices Z of order $g\times g$ with $\mathrm{Im}\,Z>0$ with respect to the modular group $\Gamma=\mathrm{Sp}(2g,\mathbf{Z})/(\pm1)$. Its Satake compactification $\overline{A_{g,1,1}\otimes\mathbf{C}}$: is known: Pieces of the boundary are the moduli varieties of Abelian varieties of lower dimension.

In a recent paper Namikawa [Y. Namikawa, Nagoya Math. J., 52, 197-259 (1973)] constructs a map from the moduli variety of stable curves into a Satake-compactified $A_{g,1,1}\otimes\mathbf{C}$ with a blown-up boundary

$$\bar{j}:\overline{M}_g\otimes\mathbf{C}\to\overline{A_{g,1,1}\otimes\mathbf{C}}.$$

It is shown that $\bar{j}$ is injective and, when g = 2, is an isomorphism. These statements are natural generalizations of Torelli's theorem.

See [515] on the moduli varieties of certain special Abelian varieties connected with polarized surfaces of type K3. A description of the boundary of the moduli space for K3 surfaces is given in [566]. See [514, 716, 717, 735, 736, 754] for other results on the moduli of algebraic varieties.

§6. Periods of Integrals on Algebraic Varieties

Historically the theory of algebraic curves arose in the works of Abel and Riemann as the analytic theory of the periods of their "Abelian" integrals, i.e., in modern terminology, as the theory of integrals of everywhere regular holomorphic differential forms of first degree with respect to cycles of one-dimensional homologies on an appropriate Riemann surface (see [122]). In such an approach at least two aims are achieved:

a) The Jacobian variety of a curve is constructed as a complex g-dimensional torus, where g is the curve's genus;

2) there arises one of the analytic methods for the construction of the moduli space of algebraic curves as the space of their complex matrices of the periods.

The analytic theory of Abelian varieties originated also as the theory of their period matrices with known bilinear Riemann—Frobenius relations picking out among all n-dimensional complex tori the algebraic ones (see [14], for example). We have known for a long time the analytic construction of the Albanese and Picard varieties as complex tori arising from a consideration of the period matrices of one-dimensional regular differential forms of type (1.0) and (0.1), respectively (see [8], for example). Recently Griffiths [381-383] (also see surveys [21, 385, 392]) developed the general theory of the periods of integrals of differential forms of any order on algebraic varieties with the aim of constructing the moduli space as a space of the periods.

1. Differential Forms on Algebraic Varieties. The general theory of regular differential forms on a complex projective algebraic variety was created by Hodge [429] (also see Weil [14]).

The fundamental conclusions of this theory are the following. Let V be an n-dimensional projective nonsingular algebraic (or even merely a compact Kählerian) variety over the complex number field **C**.

(1) For any integer r, $0 \leqslant r \leqslant n$, there exist a finite-dimensional linear space over C of complex-valued harmonic differential forms $\mathcal{H}^r(V)$ and its expansion into a direct sum of spaces of harmonic forms of type (p, q), $p \geq 0$, $q \geq 0$,

$$\mathcal{H}^r(V) = \sum_{p+q=r} \mathcal{H}^{p,q}(V),$$

and the operation of complex conjugation induces the isomorphism

$$\mathcal{H}^{p,q}(V) \simeq \overline{\mathcal{H}^{q,p}(V)} = H^r(V, \mathbf{Z}) \otimes \mathbf{C}.$$

(2) There exist the isomorphisms of de Rham, leading an exterior product into a product of cohomologies

$$\mathcal{H}^r(V) \simeq H^r(V, \mathbf{C}) = H^r(V, \mathbf{Z}) \otimes \mathbf{C},$$

and of Dolbeault

$$\mathcal{H}^{p,q}(V) \simeq H^q(V, \Omega^p),$$

where Ω^p is the sheaf of germs of holomorphic differential p-forms on V. The numbers $h^{p,q} = \dim_{\mathbf{C}} \mathcal{H}^{p,q}(V)$ are called Hodge numbers. There hold the relations:

$$h^{p,q} = h^{q,p}, \quad h^{p,q} = h^{n-q,n-p}, \quad b^r = \sum_{p+q=r} h^{p,q},$$

where b^r is the r-dimensional Betti number of variety V.

(3) Let $\omega^{1,1}$ be a closed differential form of type (1.1), corresponding to the class of hyperflat sections of variety V in $H^2(V, Z)$ (or the Kähler metric in the case of an arbitrary Kählerian variety), and L be the operator of exterior multiplication by $\omega^{1,1}$. Then there holds the isomorphism (the strong Lefschetz theorem)

$$L^{n-r}: \mathcal{H}^r(V) \simeq \mathcal{H}^{2n-r}(V),\ 0 \leqslant r < n.$$

(4) Let $\mathcal{H}^r(V)_0$ be the nucleus of the homomorphism

$$L^{n-r+1}: \mathcal{H}^r(V) \to \mathcal{H}^{2n-r+2}.$$

It is called the space of primitive r-forms. Let $\mathcal{H}^{p,q}(V)_0 = \mathcal{H}^{p,q}(V) \cap \mathcal{H}^{p+q}(V)_0$ be spaces of primitive forms of type (p, q) and let $h_0^{p,q} = \dim \mathcal{H}^{p,q}(V)_0$. There holds the Weil–Lefschetz expansion for $0 < r \leqq n$

$$\mathcal{H}^r(V) = \sum_{0 \leqslant i \leqslant [r/2]} L^i \mathcal{H}^{r-2i}(V)_0.$$

(5) There is defined the nondegenerate quadratic form

$$Q: \mathcal{H}^r(V)_0 \times \mathcal{H}^r(V)_0 \to \mathbf{C},$$

$$Q(\xi, \eta) = (-1)^{\frac{r(r+1)}{2}} \int_V (\omega^{1,1})^{n-r} \wedge \xi \wedge \eta,\quad \xi,\ \eta \in \mathcal{H}^r(V)_0$$

(symmetric or skew-symmetric depending on the parity of r) satisfying the Riemann–Hodge relations (see Griffiths [381])

$$Q\left(\mathcal{H}_0^{r-p,p},\ \mathcal{H}_0^{r-q,q}\right) = 0,\ q \neq p,$$

$$Q\left(\mathcal{H}_0^{r-p,p},\ \overline{\mathcal{H}}_0^{r-p,p}\right) > 0;$$

the latter condition signifies that $i^r(-1)^{r+p} Q(\xi, \bar{\eta})$ is a positive definite Hermitian form.

With the aid of the de Rham isomorphism all the listed properties of the spaces of harmonic forms carry over to the spaces of cohomologies $H^r(V, \mathbf{C}) = H^r(V, \mathbf{Z}) \otimes \mathbf{C}$, so that we can speak of primitive classes of cohomologies $H^{p,q}(V)_0$, of Hodge numbers $h_0^{p,q}$, of quadratic forms Q (which for algebraic varieties are defined over $\mathbf{Z}$), etc.

2. Variations of Hodge Structures. Let E be a complex vector space with conjugation $e \to \bar{e}$ $(e \in E)$. A polarized Hodge structure of weight k in the sense of Deligne [25, 209, 210] and Griffiths [21, 385] consists of:

a) the giving of the Hodge filtration $F^0 \subset F^1 \subset \ldots \subset F^k = E$, possessing the property that $F^p \oplus \bar{F}^{k-p-1} \simeq E$ is an isomorphism for any $0 \leqq p \leqq k$; this is equivalent to the giving of the expansion into a direct sum

$$E = \sum_{p+q=r} E^{p,q},\ E^{p,q} = \bar{E}^{q,p},$$

where

$$E^{p,q} = F^p \cap \bar{F}^{k-p},\ F^p = \sum_{p' < p} E^{p',q'};$$

b) the giving of a Z-lattice H, $E = H \otimes \mathbf{C}$, and of a bilinear Z-form $Q: E \times E \to \mathbf{C}$ such that

$$Q(E^{p,q},\ E^{p',q'}) = 0,\ \text{if}\ \ p \neq p',\ q \neq q',$$

and

$$i^{p-q} Q(E^{p,q},\ \bar{E}^{p,q}) > 0$$

(these are the bilinear Riemann–Hodge relations).

Let $f: X \to S$ be a proper smooth morphism of algebraic varieties over the complex number field **C**. We assume that a polarization is given on X, namely, the class of very ample invertible sheaves $L_{X/S}$. Then, on each geometric fiber in the space of primitive classes of homologies $H^k(X_S, C)$ there is defined a polarized Hodge structure of weight k, and, as Griffiths showed, there is defined a map of periods $\varphi_k: S \to M_k(X_0)$, where $X_0 = f^{-1}(s_0)$ is some fixed fiber over $s_0 \in S$, $M_k(X_0)$ is the space of k-dimensional periods of variety X_0. It is defined in the following way. Let D_k be the space of all polarized Hodge structures of weight k on $E = H^k(X_0, \mathbf{C})_0$ with a fixed collection of numbers (p, q), a Z-lattice of primitive integral cohomologies, and a form Q [this is some open subset in the algebraic variety of (incomplete) flags in E]. Then D_k is a homogeneous complex space relative to a real Lie group $G = O(E, R)$ of linear real automorphisms E preserving form Q, and the stationary subgroup $H \subset G$ is compact (H is not necessarily a maximal compact subgroup). The space $M_k(X_0)$ is the factor space of the homogeneous complex variety D_k by a discrete group of Z-automorphisms E preserving Q.

The fundamental aim of Griffiths' work [381-383, 385-387] is the study of the period spaces $M_k(X_0)$ and of the map of periods. The space $M_k(X_0)$ is called the moduli space of the Hodge structure; for each $0 < k \leqslant \dim X_0$ such a space is determined by k-dimensional primitive cohomology classes.

Griffiths [382] discovered that for varieties of dimension $n \geqslant 2$ and for $k > 0$, besides the Riemann–Hodge conditions (included in the definition of the Hodge structure), there exist some more local conditions on the period map φ_k. Let K be a maximal compact subgroup containing H in G and $R = K \setminus G$ be the corresponding symmetric Riemann space. Then in the fibering

$$\begin{array}{ccc} D_k & \simeq & H \setminus G \\ \omega \downarrow & & \downarrow \\ R & \simeq & K \setminus G \end{array}$$

there exists a unique G-invariant connection

$$T_d(D_k) = V_d \oplus H_d, \; d \in D_k,$$

separating in the tangent space T_s a vertical (tangential to the fiber) subspace V_d and a horizontal one H_d. The local conditions are that the period map is always horizontal. Group K differs from H, for example, if two numbers $0 \leqslant p_1 < p_2 \leqslant k/2$, exist such that $h_0^{p_1, k-p_1} \neq 0$, $h_0^{p_2, k-p_2} \neq 0$. For a smooth bundle Griffiths' local conditions are expressed also (see [21, 385]) in terms of the Gauss–Manin connection arising from the natural action of the fundamental group $\pi_1(S, s_0)$ in the cohomologies $H^k(X_0, C)$.

In the study of period maps there arise the following fundamental problems which are discussed in detail in Griffiths' survey [21, 385].

1) A circle of problems connected with the generalization of Torelli's theorem. The local Torelli problem consists in the following: when the Hodge structure on $H^*(X_0, C)$ separates the points in the local moduli space (the Kuranishi space) of variety X_0. The global Torelli problem: when the polarized Hodge structure on $H^*(X_S, C)$ completely determines the polarized algebraic variety X_S.

As Griffiths [21, 383, 385] noted, the local Torelli theorem is valid in many cases: for curves, hypersurfaces in P^n (except cubic surfaces), hypersurfaces in Abelian varieties, and in varieties with a trivial canonic sheaf.

Kii [40] showed that it is true for varieties which are cyclic coverings of P^n and possess positive canonic divisor classes.

The global Torelli theorem (as far as the authors know) has been proved only in the following cases (besides the usual Torelli theorem for curves):

for polarized surfaces of type K3 by Pyatetskii-Shapiro and Shafarevich [96];

for a nonsingular cubic hypersurface in $\mathbf{P}^4(C)$ by Tyurin [106, 107] and by Clemens and Griffiths [41, 205];

for a nonsingular intersection of two quadrics in $P^N(C)$ in Rid's dissertation (in press); for N = 4 this was established in [643] as well.

Interesting global Torelli theorems have been obtained for the moduli varieties of stable vector bundles in [109, 110, 592].

2) Problems connected with compactification and with the behavior of the periods at infinity. Let

$$\begin{array}{ccc} X & \subset & \bar{X} \\ f\downarrow & & \downarrow \bar{f} \\ S & \subset & \bar{S} \end{array}$$

be the compactification of a smooth morphism f; here $\bar{X}$, $\bar{S}$ are smooth complete varieties, $\bar{X} - X$ and $\bar{S} - S$ are divisors with normal intersections. Let $\varphi_k: S \to M_k(X_0)$ be a period map. Does a reasonable compactification $\overline{M_k(X_0)}$ exist and, if it does, can the map φ_k be continued upto the map $\bar{\varphi}_k: \bar{S} \to \overline{M_k(X_0)}$? An affirmative answer to this question has been obtained only for curves and partially for a certain class of surfaces (see Griffiths [21, 385]). Interesting profound results on that score exist in Schmid [681].

3) Other problems and results connected with local and global monodromies (see §4), as well as with the theory of uniformization and with automorphic forms for monodromy groups.

In [382] Griffiths introduces the very important concept of an intermediate Jacobian. Let X be an n-dimensional projective variety over C. For any $0 \leqslant q < n$ we set $H_+^{2q+1}(X) = H^{2q+1,0}(X) + \ldots + H^{q+1,q}(X)$. Then the factor space

$$T_q(X) = H_+^{2q+1}(X) \diagdown H^{2q+1}(X, \mathbf{C})/H^{2q+1}(X, \mathbf{Z})$$

is a complex torus and is called the intermediate Jacobian of Griffiths. As Griffiths showed [382], this torus can be holomorphically varied under a holomorphic variation of variety X. It is not always an Abelian variety, in contrast with the intermediate Jacobian of Weil. The latter does not possess the property of holomorphicity under variations. Griffiths' Jacobians $T_q(X)$ play an important role in the theory of algebraic cycles on variety X (see §3). When q = 0 the torus $T_0(X)$ is the Picard variety of variety X; when q = n–1, $T_{n-1}(X)$ is the Albanese variety.

3. Hodge–Deligne Theory. In [25, 209, 210] Deligne develops a certain algebraic analog of Hodge theory for nonsingular and not necessarily complete algebraic varieties over C. This, as Deligne named it, is a mixed Hodge theory. Let X be a smooth quasiprojective algebraic variety over C. According to Hironaka's results on the resolution of singularities there exists a smooth compactification $\bar{X}$ of variety X, being a projective variety over C, and $Y = \bar{X} - X$ is a union of smooth divisors with normal intersections. By $\Omega^p_{\bar{X}}\langle Y\rangle$ we denote the sheaf of differential p-forms on $\bar{X}$ having no more than "logarithmic singularities" on Y (i.e., poles of no higher than first order) and not having singularities on X. Let $\Omega^*_{\bar{X}}\langle Y\rangle$ be the corresponding de Rham logarithmic complex and $\mathbf{H}^*(\bar{X}, \Omega^*_{\bar{X}}\langle Y\rangle)$ be hypercohomology. Deligne shows that the isomorphism

$$H^*(X, \mathbf{C}) \simeq \mathbf{H}^*(\bar{X}, \Omega^*_{\bar{X}}\langle Y\rangle)$$

holds and defines two filtrations on $\Omega^*_{\bar{X}}\langle Y\rangle$: an increasing filtration of weights $W_n(\Omega^*_{\bar{X}}\langle Y\rangle)$ and the Hodge filtration F^k. These filtrations generate two spectral sequences, converging to $H^*(X, \mathbf{C})$, and induce on the cohomologies a mixed Hodge structure consisting of a choice of the lattice $H = H^*(X, \mathbf{Z}) \subset H^*(X, \mathbf{C})$, a filtration W_n on $H \otimes \mathbf{Q}$ and a filtration F on $H \otimes \mathbf{C}$, not depending upon the choice of the compactification of variety X.

If X is a complete smooth variety, then the mixed structure reduces to the usual Hodge structure on $H^*(X, C)$ in which $W_n(H^*(X, \mathbf{C}))=H^n(X, \mathbf{C})$.

Now let $f\colon X \to S$ be a projective smooth morphism over C; then, as was shown in [207], the Leray spectral sequence

$$E_2^{p,q}=H^p(S, R^q f_* \mathbf{Q}) \Rightarrow H^{p+q}(X, \mathbf{Q})$$

degenerates (i.e., $E_2 = E_\infty$). This fact is generalized in [210] to the case when f is not necessarily projective, but only proper, and S is a smooth separable scheme of finite type over **C**.

From this result and from the mixed Hodge theory on $H^*(X, C)$ Deligne derives a theorem on the complete reducibility of a global monodromy group, Griffiths' theorem [383] on the preservation of type (p, q) of the Hodge component under variations, Grothendieck's rigidity for Abelian schemes, and others.

Deligne's theory of weight filtration and of mixed Hodge structures proved to be very useful in the study of the compactification of the period space and of the period maps at infinity in Griffiths' theory. In [21] Griffiths gives a precise formulation of Deligne's conjecture on this score, concluding, roughly speaking, that on the boundary the mixed Hodge structure on cohomologies is the limit of the usual Hodge structure on the cohomologies of nonsingular fibers. The latter is directly connected with the monodromy transformation around the components of the boundary. Deligne's conjecture has been verified in certain special cases (see [21, 681]).

§7. Geometry of Algebraic Curves

The fundamental results in the geometric theory of algebraic curves were obtained as far back as the past century in the works of the German mathematicians Riemann, M. Noether, A. Brill, as well as of Italian geometers. The reorganization of the fundamentals of algebraic geometry also affected the theory of algebraic curves. Many classical results were re-proved by modern means, while a part of them were significantly strengthened. See [257, 697] for new textbooks on the theory of algebraic curves.

1. Special Divisors. One of the important questions in the geometry of algebraic curves is that of the existence of a complete linear system $|D|$ of divisors of a given degree $d > 0$ with $\dim|D|\geqslant r\geqslant 0$ on a smooth complete curve of genus g. A very weak statement on that score is the Riemann–Roch theorem asserting that $\dim|D| = \dim|K-D|+d-g+1$. It gives a necessary condition for an affirmative answer to the preceding question: $r-d+g\geqslant 0$. If a divisor D is not special (i.e., $|K-D| = \varnothing$; for example, $d>2g-2$), then the preceding inequality turns into an equality and, consequently, when $r>d-g$ there can be an affirmative answer only for a special divisor D. Riemann discovered that the answer is in the affirmative if $\tau = (r+1)(d-r)-rg \geqslant 0$; in addition, the set of such divisors must depend upon $\tau + r$ parameters. T. Meis and Gunning [404] gave an analytic proof of this statement for the case $r = 1$. An algebraic proof (true for an arbitrary algebraically closed ground field) was given by Kempf [481] in the general case and independently by Kleiman and Laksov [491]. Both these proofs use the Porteous formula (see [640]) defining the class of rational equivalence of a certain determinant Jacobian subvariety of the variety of the curve. The technique used is based chiefly on the papers of Mattuck [563] and Schwarzenberger [683]. Paper [728] is devoted to a survey of this work.

Mayer [565] gave a rigorous proof of the following statement of Riemann: Let Θ be a Poincaré divisor on the Jacobian variety J of a curve X of genus g, representing complete linear systems of degree $g - 1$ on X. Then the projective dimension of the corresponding system is one less than the multiplicity of the corresponding point on Θ. In particular, if Θ_n^r denotes the set of points on J, corresponding to complete linear systems of degree n and of projective dimension $\geqslant r$, then a corollary of this statement is the fact that the points of Θ_{g-1}^1 are all singular points of the Poincaré divisor $\Theta=\Theta_{g-1}^0$. According to a result of Weil, for any $n \leqslant g-1$, Θ_n^1 is the set of singularities of the subvariety Θ_n^0 for a general (in the sense of the moduli variety) curve X. Using the results in [248], Martens [549] showed that the presence of singular points in $\Theta_n^1 \setminus \Theta_n^2$ for $n \leqslant g-1$ is special in the sense of moduli theory. This same paper gives a rigorous proof of the well-known Clifford theorem: If $\deg(D) \leqslant g-1$ and d $\neq$ 0, then $l(D) = \dim|D|+1 < (\deg(D)+1)/2$.

By generalizing Riemann's result, Kempf determined in [482] an ideal giving the tangent cone at a singular point of variety Θ_n^0 $(n \leqslant g-1)$. For n = g − 1 this result was obtained by Mumford (unpublished).

The theory of special divisors is closely connected with the theory of theta functions. The variety Θ_{g-1}^0 above is a shift of the divisor of the zeros of the theta-function Θ, while the equation of the tangent cone at the point $\omega \in \Theta_{g-1}^0$ is the principal term in the expansion of Θ in a power series at point ω. See Fay's book [251] in regard to this connection.

2. Automorphisms of Curves. The classical Schwartz–Wiman theorem states that the automorphism group of a curve X of genus g > 1 is finite. In case $g \leqslant 1$ this group is infinite; for g = 1 it is described in the theory of elliptic curves; the case g = 0 is trivial. Hurwitz's theorem gives a bound for the order of the group Aut(X) of a curve X of genus g > 1: $|\mathrm{Aut}(X)| \leqslant 84(g-1)$. From the proof of this theorem (see [120], for example) it follows easily that the bound is achieved only for curves X such that $X/\mathrm{Aut}(X) \simeq \mathbf{P}^1$, while the natural projection $X \to \mathbf{P}^1$ is a Galois covering with a branching scheme (2,3,7). An example of such curves is the Klein curve of genus 3 (whose canonic model is given by the equation $x_0^3x_1+x_1^3x_2+x_2^3x_0=0$) and the Macbeath curve of genus 7 [240, 539], as well as their maximal Abelian unramified coverings of genus $m^{2g}(g-1)+1$ (g = 3 or 7).

A lower bound for the maximal order $\mu(g)$ of the automorphism group of a curve of genus g was given by Maclachlan [540] and Accola [131]. They showed that $\mu(g) \geqslant 8(g+1)$ and that this bound is exact for infinitely many values of g. See [483] for a refinement of these results.

Interesting examples of curves with computed automorphism groups have been pointed out by Edge [241, 242]. Modular curves furnish many examples of a computed automorphism group.

The possible orders of the elements of the automorphism group of a curve were indicated in [415].

It is very well known that nonhyperelliptic curves of genus g > 0 with a nontrivial automorphism group are the singular points of the corresponding moduli variety M_g of curves of genus g. Interesting results connected with this fact have been obtained by Popp [634-636] and Rauch [649]. Let T_g be a Torelli space of curves of genus g [649], Ω be the discrete automorphism group of T_g, anti-isomorphic to group Sp(2g, Z), $\Omega(l)$ be its principal congruence-subgroup of level l. For $l \geqslant 3$ the factor $M_g(l) \doteq T_g/\Omega(l)$ is a smooth quasiprojective algebraic variety of dimension 3g–3 on which the finite group $G = \Omega/\Omega(l)$ acts. The factor $M_g(l)/G = T_g/\Omega$ is the "crude" moduli variety M_g. For any point $x \in M_g$ the inertia group I(x') of any point x' lying over x under the canonic projection $M_g(l) \to M_g$, is isomorphic with the automorphism group of an appropriate curve. In particular, for a given g there exists only a finite number of automorphisms of curves of genus g [since they are all subgroups of the finite group $G = \Omega/\Omega(l)$]. The whole variety is divided into a finite number of nonintersecting smooth locally closed subvarieties W_Γ consisting of points representing curves with an automorphism group isomorphic with a given group Γ. When a curve X belongs to a zero-dimensional subvariety W_Γ, we say that X has many automorphisms. When $g \geqslant 4$ the corresponding points in M_g are isolated singular points. The Klein and the Macbeath curves are examples of curves with many automorphisms [635].

3. Projective Immersions of Curves. The Riemann–Roch theorem on a smooth complete algebraic curve X shows that any invertible sheaf $\mathscr{L}$ on X of positive degree is ample. If $X \simeq \mathbf{P}^1$, then it is very ample; if X is an elliptic curve, then $\mathscr{L}^{\otimes 3}$ is always very ample. For a curve X of genus $g > 1$ the sheaf $\mathscr{L}$ with degree $\deg(\mathscr{L}) \geqslant 2g+1$ is very ample. Here the image of X relative to a complete linear system defined by sheaf $\mathscr{L}$, can always be given by equations of second or third degree [675, 676]. The condition $\deg(\mathscr{L}) \geqslant 2g+1$ is not, it is clear, necessary for very ampleness. For example, if X is a nonhyperelliptic curve, then the canonic sheaf ω_X of degree $2g - 2$ is very ample; for an elliptic curve $\omega_X^{\otimes 2}$ is very ample). The image of X relative to the canonic embedding, with explicitly listed exceptions, can be given by second-degree equations. This classical Enriques–Noether–Petri assertion was re-proved by modern techniques by Shokurov [127] and Saint-Donat [677].

A curious fact is that each curve with $g \leqslant 5$ can be immersed into some K3-surface (see [566], for example). Tyurin states (unpublished) that this is so for $g \leq 11$. Hence follows the unirationality of the moduli variety of curves of genus $g \leq 11$ (see §5).

4. One-Half of the Canonic Class. Simple arguments in the theory of algebraic curves show that the set of divisors D on a curve X of genus $g > 0$, such that $2D \sim K_X$, forms a principal homogeneous space S(X) relative to the group J_2 of second-order points on the Jacobian variety J of curve X. In particular, S(X) consists of 2^{2g} elements (called, in the classical literature, the "theta-characteristics" of curve X). The fundamental classical results on S(X), going back to Riemann and Wirtinger and based on the theory of theta functions, have been re-proved and generalized recently by Atiyah [145] (for case k = C) and by Mumford [588] ($\mathrm{char}\, k \neq 2$).

Using the general theory of spinor varieties, the first one of them proves the following statement of Riemann: If D_t is a holomorphic family of theta-characteristics on a family of curves X_t, then $\dim|D_t|$ is constant modulo 2. The function $\varphi: S(X) \to \mathbf{F}_2$, defined by the equality $\varphi(D) = \dim|D| + 1$, is a quadratic form corresponding to the bilinear form which is the $\cup$-product on the space of cohomologies $H^1(X, \mathbf{Z}/2)$. The number of zeros of this function equals $2^{2g-1} + 2^{g-1}$ (hence follows, for example, such a beautiful fact: The number of bitangents to a flat nonsingular quartic equals 28). The connection with spinor varieties is explained by the fact that the set of theta-characteristics is found to be in bijective correspondence with the set of spinor structures on an appropriate Riemann surface.

In [588] Mumford generalizes the concept of theta-characteristics to bundles of arbitrary rank, by examining the bundles E possessing the quadratic form $E \otimes E \to \Omega_X^1$. For families of such bundles he proves that $\dim P(X, E_t)$ is constant modulo 2. Using the results in [583], he carries over the rest of Riemann's statements to the case of characteristic $\neq 2$. Here the group of cohomologies $H^1(X, Z/2)$ is replaced by the group of étale cohomologies $H^1(X_{\mathrm{et}}, \mu_2)$.

§8. Geometry of Algebraic Surfaces

The reorganization, beginning in the Fifties, of the foundations of algebraic geometry on the basis of modern techniques of homological and commutative algebra necessitated a critical reconsideration of the majority of the classical results in the theory of surfaces. A part of them, relating to the foundations of the theory, were generalized to varieties of higher dimension (Riemann–Roch theorem, Picard varieties, etc.). Many results, specific to the case of surfaces, were re-proved anew with the use of new ideas and methods, being here partially revised and supplemented. However, we must remark that the influence of the old ideas and methods was obvious in many papers. See [2, 49, 581, 756, 757] for monographs and surveys on the theory of surfaces.

1. Classification of Algebraic Surfaces. One of the most brilliant achievements of the Italian school of geometers is the classification of algebraic surfaces, obtained mainly by Enriques. According to his results, every nonsingular projective algebraic surface over the complex number field belongs to one of the following six classes which are characterized by the values of the numerical invariants of the surfaces occurring in them:

1) Rational surfaces (birationally isomorphic with the projective plane P^2): $p_a = p_2 = 0$.

2) Ruled surfaces (birationally isomorphic with the product $P^1 \times C$, where C is an algebraic curve of genus $g > 0$): $P_{12} = 0$.

3) Elliptic surfaces (possessing a pencil of elliptic curves) whose minimal models (other than P^2) are characterized by the condition: $(K^2) = 0$ and $P_{12} > 1$ or $p_g = 0$ and $P_{12} = 1$.

4) Two-dimensional Abelian varieties: $K = 0$ and $p_a = -1$.

5) K3-surfaces (birationally isomorphic to a surface with $K = 0$, $p_a = 1$).

6) Surfaces of fundamental type (with $\varkappa = 2$; see §1) whose minimal models are characterized by the condition $(K^2) > 0$, $P_2 \geqslant 2$.

Here (and everywhere in what follows) for any smooth projective surface X over C the invariants p_a, p_g, P_n are defined as in §1.

A characterization of rational surfaces (a rationality criterion) was obtained already in 1955 by Kodaira (see [707]). At the start of the Sixties a complete restoration of the Enriques classification was made in the papers of Kodaira [501-510] at the seminars of Shafarevich in Moscow [2] and of Zariski at Harvard. Here the greater part of the proofs were essentially algebraic and used individual results carried over to the case of an arbitrary field of characteristic zero. Zariski [760] carried the rationality criterion to the case of arbitrary characteristic.

Certain results in the direction of a general classification of algebraic surfaces over fields of positive characteristic were obtained by Mumford. He noted that the list of possible classes must be enlarged (for example, the class of quasielliptic surfaces exists). Most difficult, it seems, is the proof of an analog of the ruledness criterion: $P_{24} = 0$ (this formulation was suggested by Shafarevich).

A more detailed study of each of the classes of algebraic surfaces was made by many authors. Thus, minimal models of ruled surfaces were investigated by Nagata [83, 84, 598, 599]. A classification of elliptic surfaces was carried out by Shafarevich using the theory, created by him (and independently by Ogg [621]), of smooth homogeneous spaces of algebraic varieties over functional fields (see [2, 121]). An analytic variant of this theory was obtained independently by Kodaira [503, 504]. Some of the results obtained here were predicted (often not exactly) by Enriques. Kodaira [505], Tyurina [2, 114], Andreotti, Weil (cf. [505]) studied the moduli variety and the topology of surfaces of type K3. Shafarevich and Pyatetskii-Shapiro [96] proved a global Torelli theorem (see §6) for such surfaces. They [95] (and, independently, Deligne [220]) verified the Riemann conjecture for this class of surfaces (although the main result of these papers overlaps a recent proof by Deligne of the Riemann-Weil conjecture in the general case, the methods developed in them are of independent interest). Saint-Donat (preprint) has shown that besides the explicitly listed cases, K3-surfaces can be given by equations of second degree. Special K3-surfaces (i.e., having hyperelliptic curves) were studied in [1, 32]. In particular, the explicit equations were indicated, yielding a special K3-surface in the form of a double plane. The geometry of Kumer surfaces is studied in [96].

For surfaces of fundamental type Moishezon has shown that $|9K|$ determines a birational morphism in a projective space. Later Kodaira [499, 500] strengthened this result — the same is true for $|6K|$. The definitive result in this direction is due to Bombieri [170, 171]: $|5K|$ always yields a birational morphism and surfaces exist for which $|4K|$ does not possess this property [for such surfaces necessarily $(K^2) = 1$].

There exist distinguished classes of surfaces, different from those indicated in the Enriques classification. For example, the class of hyperelliptic surfaces (surfaces representable in the form of a factor of a two-dimensional Abelian variety) is interesting. A part of such surfaces fall into the class of elliptic surfaces, while the rest are either K3-surfaces or ruled surfaces. A portion of the extensive Enriques—Severi memoir [244], relating to elliptic hyperelliptic surfaces, has been reproduced recently in Suwa [725, 726].

The Enriques surface, characterized by the conditions $p_a = p_g = 0$, $2K = 0$, has been well studied (see [1, 2]). In case the ground field is of characteristic zero all such surfaces are elliptic [2]. The authors do not know if this is true in the general case.

2. Constructions of Surfaces. The problem of the existence of an algebraic surface of given type with specified numerical (or other) invariants remains one of the most interesting unsolved problems in the theory of surfaces. Besides the equality $1+p_a = [(K^2) + c_2]/12$ (Noether's formula), no other necessary relations whatsoever are known between the arithmetic genus and the Chern classes of the surface's minimal model. For example, the question of the existence of simply connected algebraic surfaces of fundamental type with $p_a = 0$ remains open. Irrational elliptic surfaces with such a property were constructed in [28]. A result was announced in [274] (there are gaps in the proof) that such surfaces do not exist if $(K^2) > 1$ and the bicanonic system is irreducible. Regular surfaces of fundamental type with $3 \leqslant (K^2) \leqslant 8p_a + 7$ were constructed by Burniat [190] (there are a number of incomprehensible places in the proof). Interesting and important necessary relations between (K^2) and c_2 were given by Van de Ven [744]. The bound $(K^2) \leqslant \max[8c_2, 2c_2]$ was proved.

The inequality $1+p_a \leqslant (1/2)(K^2)+3$ is always fulfilled for surfaces of fundamental type (see [171]).

A large number of explicit constructions of surfaces with specified concrete invariants were given by Godeaux [272-322] (also see [741-745]).

Some general methods for an explicit construction of surfaces are known. One of them is the method of constructing the surface in the form of a ramified covering of some known surface (for example, in the form of a double plane). The invariants of such a surface are computed in terms of the invariants of the branching curve (see [462], for example). Another method is, in a certain sense, the opposite of the first. The desired surface is constructed in the form of a nonsingular model of the factor of some known surface by a finite (for example, cyclic) automorphism group. Many papers [269, 308-312] of Godeaux were based on this method. As an instance, one of the first examples of surfaces of general type with $p_a = p_g = 0$ (a Godeaux surface; see [170]) was constructed by this method.

A number of interesting examples of algebraic surfaces is connected with the theory of automorphic functions (see [424-427, 718, 719]).

3. Automorphisms of Algebraic Surfaces. The group Aut(X) of a smooth complete algebraic surface X has the structure of the scheme of groups of locally finite type, whose connected component of unity $\mathrm{Aut}(X)^{\circ}$ is an algebraic group. It was studied individually in each of the six classes in the Enriques classification. Here, obviously, we can take it that the surface is a minimal model.

In the case of rational and ruled surfaces the group $\mathrm{Aut}(X)^{\circ}$ is an algebraic group containing a linear projective group (see [551]).

If X is an elliptic surface, then $\mathrm{Aut}(X)^{\circ}$ can be nontrivial (in this case this is an Abelian variety and the Jacobian surface for X is a trivial bundle). The group of connected components of $\mathrm{Aut}(X)/\mathrm{Aut}(X)^{\circ}$ can be both a finite as well as an infinite group. These facts and more detailed information on Aut(X) can be obtained from Shafarevich's classification of elliptic surfaces (see [2]).

The automorphism group of an Abelian surface is described on the basis of the general theory of Abelian varieties (see [50]).

The problem of computing the automorphism group of a K3-surface is of interest. In case $k = \mathbf{C}$ the general results in this direction were obtained by Shafarevich and Pyatetskii-Shapiro in [96]. The group $\mathrm{Aut}(X)^{\circ}$ is always trivial, while Aut(X) is isomorphic to within a finite group to the factor group of the automorphism group of an integral quadratic form defined on NS(X) by the intersection index, by the subgroup generated by the maps. Examples exist (some of them were known at the beginning of the century) of surfaces withboth an infinite as well as a finite automorphism group (see [96, 320]). The automorphism group of special K3-surfaces has been computed by Dolgachev (unpublished). Of interest is the problem of extending the Shafarevich—Pyatetskii-Shapiro theorem to the case of a ground field of arbitrary characteristic (the triviality of $\mathrm{Aut}(X)^{\circ}$ is obvious in the general case).

The automorphism group of surfaces of fundamental type is always finite. In contrast to the case of algebraic curves (see §7) there are apparently no known general assertions whatsoever on its construction (the authors do not even know of any nontrivial examples of its computation). A very excessive bound exists for its order in terms of the numerical. invariants of the surface [A. Andreotti, Rend. Mat. e Appl., 9, No. 314, 255-280 (1950)].

Shafarevich's paper [716], in which the automorphism group of an affine plane was computed, serves as the start of the investigations of the automorphism group of affine surfaces. Danilov [24] has proved the nonsimplicity of its unimodular subgroup. An affine plane is homogeneous relative to its own automorphism group. A cycle of papers [16-20] by Gizatullin was devoted to the classification of smooth affine surfaces quasihomogeneous (i.e., homogeneous outside of a finite set of points) relative to their own automorphism groups. A part of them are quasihomogeneous already for a certain algebraic automorphism group [19]. The latter surfaces (not necessarily smooth) were found by Popov [90, 91] on the basis of the theory of algebraic transformation groups.

4. Zero-Dimensional Cycles. One of the old unsolved problems in the theory of surfaces is the following problem of Severi. Is it true that rational surfaces are characterized by the condition that each zero-dimensional cycle of degree zero is rationally equivalent to zero? A number of papers by Severi himself, devoted to the study of the class group $A_0(X)$ of zero-dimensional cycles, remain incomprehensible to this day and, as Mumford [56] showed, a part of them are erroneous. For example, Severi's assertion that $A_0(X)$ implies the simple-connectedness of X and the equality to zero of $p_a(X)$ has not been proved till now. In contrast to what Severi expected there do exist irrational surfaces with such a property [28]. Mumford [56] proved that if $p_g > 0$ the group $A_0(X)$ is in some sense of infinite type. This result was generalized to higher-dimensional varieties by Roitman [97, 98].

§9. Vector Bundles

The concept of a vector bundle arrived into algebraic geometry via topology and analytic geometry of varieties in the Fifties of our century [119]. It was used to clear up the concept of characteristic classes and to profoundly generalize the Riemann–Roch theorem in the papers of Hirzebruch and Grothendieck (see §2). It lies at the base of Grothendieck K-theory and plays an important role in algebra thanks to the connection Serre established between it and the concept of a projective module, and in the arithmetic of algebraic varieties.

Below we shall go into only the geometric side of the theory of vector bundles, a central place in which is occupied by the natural problem of classifying vector bundles on algebraic varieties.

The comparison of a vector bundle E on X with the germ sheaf $\mathscr{E}$ of its sections realizes the equivalence of the category of vector bundles and the category of locally free sheaves of O_X-modules (with preservation of rank), so that instead of bundles we can talk about sheaves, and vice versa. All the most extended operations of linear spaces carry over to bundles: direct sums, tensor and exterior products, the taking of the dual, etc. The set of classes, to within the isomorphism of vector bundles of rank n on an algebraic variety X, can be interpreted cohomologically as $H^1[X, GL_n(O_X)]$, where $GL_n(O_X)$ is the sheaf of germs of continuous maps of X into the general linear group GL_n. For n = 1 this set is provided with the structure of a commutative group with the operation $\otimes$ and is called the Picard group Pic(X) of the classes of linear bundles or of invertible sheaves, and also the group of classes of Cartier divisors. If X is a complete variety, there holds the group extension

$$0 \to \mathrm{Pic}^0(X) \to \mathrm{Pic}(X) \to \mathrm{NS}(X) \to 0,$$

where $\mathrm{Pic}^0(X)$ is the group of points of an Abelian variety (the Picard variety), while NS(X) is a finitely generated Abelian (Néron–Severi) group. This yields a complete classification of linear bundles on X.

Bundles of rank $n \geqslant 2$ can be interpreted also as classes of matrices of Weil divisors, introduced by him as long ago as 1938.

For a variety X over the complex number field C we can construct an algebraic vector bundle over X with respect to each representation of the fundamental group $\pi(X) \to GL_n$. Such bundles are distinguished by the condition of existence of an integrable connection and are said to be flat (see [208]).

1. Vector Bundles on Algebraic Curves. Let X be a complete smooth curve of genus g over a field k. Then, first of all, $\mathrm{Pic}^0(X)$ is the group of k-points of the g-dimensional Jacobian variety J(X) of the curve, while $\mathrm{NS}(X) \simeq Z$; moreover, the homomorphism $\mathrm{Pic}(X) \to Z$ is a comparison of $L \in \mathrm{Pic}(X)$ with its degree, i.e., the degree of the corresponding divisor. All

bundles of rank 1 of degree d form a g-dimensional variety, namely, a smooth homogeneous space of Jacobian J(X). If k = C, the bundle L of rank 1 is flat if and only if deg(L) = 0 [85, 86].

The classification of vector bundles of rank $n > 1$ on complete curves has progressed sufficiently far mainly over the complex number field. Some facts remain true over any algebraically closed field of characteristic 0 [712]. Only individual results are known in characteristic $p > 0$ [710, 712].

In 1957 Grothendieck gave a classification of vector bundles on the projective straight line $\mathbf{P}^1_{\mathbf{C}}$. He showed that any vector bundle of rank n is representable (moreover, uniquely) as a direct sum of linear bundles. Each linear bundle L in this case is uniquely determined by an integer, namely, its own degree.

In that same year Atiyah classified all vector bundles on an elliptic curve X over C. Now, starting with this case, in view of the nontriviality of J(X), we should introduce an invariant, namely, the determinant of bundle E of rank n (the linear bundle $\det E = \overset{n}{\wedge} E$). The determinant's degree deg(det E) is called the degree of bundle E. Atiyah showed that on an elliptic curve X, for each $n \geqslant 2$ and linear bundle L there exists only a finite number of bundles E of rank n with determinant L, which cannot be represented as a sum of bundles of lower rank. They are all obtained from one of them by a tensor product on some linear bundle M with $M^{\otimes n} = 0$.

The most profound results in the direction of classifying vector bundles on curves of genus $g \geq 2$ over C were obtained by Newstead [612-615] (also see Mumford and Newstead [592]), Tyurin [109-113], Narasimhan and Ramanan [603, 604], Narasimhan and Seshadri [605-607], Remanan [643, 644], and Seshadri [710-713]. A concept, introduced by Mumford in 1962, of a stable bundle, permitting the selection of a component of maximal dimension in the space of all classes, proved to be very useful here. A bundle E on a complete smooth curve of genus $g \geqslant 2$ is said to be stable if the inequality

$$\frac{\deg F}{\operatorname{rk} F} < \frac{\deg E}{\operatorname{rk} E}$$

is fulfilled for any proper subbundle $F \subset E$. If the sign $<$ here is replaced by the sign $\leqslant$, we obtain the definition of a semistable bundle, introduced by Seshadri [126, 711]. Mumford [578] and Seshadri [711] showed that the set (of classes to within isomorphism) of stable bundles S(X, n, d) on a curve X over C of rank n and degree d is provided with the structure of a smooth quasiprojective variety of dimension $n^2(g-1)+1$ and even of a smooth projective one if n and d are relatively prime. It is fibered in a natural fashion over the Jacobian, or rather, over its principal homogeneous space J^d, and the variety $S_L(n, d)$ of stable bundles with a fixed determinant L is the fiber. Any others are obtained from stable ones by extensions.

The map $E \to E \otimes M$, where M is any one-dimensional bundle, establishes the isomorphism

$$S_L(n, d) \simeq S_{L \otimes M^n}(n, d + n \deg M),$$

therefore, $S_L(n, d)$ depends only on n and on the residue class of d modulo n.

Let us note the most important properties of $S_L(n, d)$.

1. If (n, d) = 1, then a universal bundle exists on $S_L(n, d) \times X$ (Seshadri [711], Ramanan [643]); if $(n, d) \neq 1$, then such a bundle does not exist (Newstead [612], Narasimhan and Ramanan [643]). In [612] it was shown that even topological obstructions exist.

2. The variety $S_L(n, d)$ is unirational (Tyurin [111, 112], Newstead [613]) and even rational (Newstead [613, preprint]). If g = 2, then $S_L(2, 1)$ is the intersection of two quadrics in $\mathbf{P}^5$ (Newstead [613]).

3. If $(n, d) \neq 1$, S(X, n, d) admits of a natural compactification. The corresponding projective normal variety $\overline{S}(X, n, d)$ consists of all the so-called S-equivalent semistable bundles (Seshadri [711]). Narasimhan and Ramanan [604] showed that the nonsingular point in $\overline{S}(X, n, d)$ correspond precisely to stable bundles, except the case g = 2, n = 2, d = 0. In

the latter case $\overline{S}(X, 2, 0)$ is nonsingular and is represented as a bundle over the Jacobian with fiber P^3. Narasimhan and Seshadri [605-607] have shown that stable bundles of degree 0 correspond to classes of equivalent irreducible representations of the fundamental group. Hence we can obtain a certain compactification of $S(X, n, 0)$; having added the classes of reducible ones, it coincides with the preceding one. In case $d \neq 0$, representations of certain groups of Fuks type are examined (Narasimhan and Ramanan [85]).

4. If $(n, d) = 1$, $n \geqslant 2$, then $\mathrm{Pic}[S_L(n, d)] = Z$ and the canonic class of $S_L(n, d)$ equals $-2u$, where $u \in \mathrm{Pic}$ is an ample generator [604]. There holds the isomorphism $J_3[S_L(n, d)] \simeq J(X)$, where J_3 is the three-dimensional intermediate Weil Jacobian [Narasimhan and Ramanan (in press), Mumford and Newstead [952], Tyurin (in press)].

The group $\mathrm{Aut}[S_L(n, d)]$ is finite.

5. For a curve X there hold different analogs of Torelli's theorem, connected with bundles and rank $n \geqslant 2$ (Tyurin [110, 111], and also [592, 643]). The strongest of them is: e Curve X is uniquely determined by the variety $S(X, n, d)$ if $(n, d) = 1$; in particular, theiri moduli varieties are isomorphic [Narasimhan and Ramanan (in press), Tyurin (in press)].

Seshadri [710, 712] extended the existence theorem for the moduli variety of stable bundles to an algebraically closed field of any characteristic.

Individual results on the properties of vector bundles on curves are in [268, 619, 731, 735], in particular, for characteristic $p > 0$.

2. Vector Bundles on Varieties of Dimension $n \geqslant 2$. Here as yet there is no single theory whatever. Shafarevich [684-687] (also see [688]) studies unfactorable vector bundles of rank 2 on the projective plane, and also on any projective surface. It is shown, in particular, that with the aid of monoidal transformations any bundle on a surface can be represented as a chain of extensions of linear bundles. As we have already mentioned (see §1), an analogous fact was proved by Hironaka [116] for any projective smooth varieties in characteristic 0. We remark that on curves any bundle is always representable in such a form (see [109], for example).

Hartshorne [411] introduced the concept of an ample bundle of any rank. See survey [31] regarding his results and also those connected with them. Properties of Chern classes for vector bundles are studied in [413, 414, 486, 755]; in particular, the relation $0 < c_2 < c_1^2$ for ample bundles on a nonsingular projective surface is established in [486].

Takemoto [729] introduces the concept of a stable vector bundle on an algebraic surface and proves, under certain restrictions, the existence of a moduli scheme of finite type for bundles of rank 2 with fixed Chern classes.

Oda [620] studies vector bundles of rank 2 on a two-dimensional Abelian variety over $\mathbf{C}$. It is shown that unfactorable bundles exist not representable as the direct image of one-dimensional ones with isogenies of degree 2. From the vanishing of all Chern classes does not, in general, follow the existence of an algebraic or holomorphic connection.

Weil's results on flat bundles are generalized to arbitrary algebraic complex varieties in [444].

Certain results connected with Serre's problem on vector bundles on affine spaces can be found in [595].

Papers [432, 433, 721, 740] are devoted to the study of vector bundles on P_C^n. Horrocks, in particular, studies the continuation of vector bundles with P_C^N onto P_C^n, $n \geqslant N$. An interesting example of an unfactorable bundle of rank 2 on P_C^4 was constructed by Horrocks and Mumford [435]. The zeros of its sections yield all the Abelian surfaces in P_C^4. A conjecture is made that unfactorable two-dimensional bundles do not exist on P_C^n for sufficiently large n.

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