# MATH 296 PROBLEMS 13

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**Regular problems:**

**1.** (a) Find the derivative of

$$F(x) = \int_a^{x^3} \sin^3 t dt.$$

(b) Find the derivative of

$$F(y) = \int_0^y x \sin(y) dx.$$

**2.** (a) Calculate the length of the segment of the parabola $y = x^2$ between $x = 0$ and $x = a$.

(b) Calculate the length of the "spiral curve" in $\mathbb{R}^3$ parametrized by

$$(t, \cos(t), -\sin(t))$$

for $t \in [0, 2\pi]$.

**3.** Calculate the area of the hyperboloid, i.e. surface obtained by rotating the graph of $y = 1/x$, $x \in [1, 2]$ around the $x$ axis.

**4.** Calculate

$$\lim_{n\to\infty} \left( \frac{1}{n^2} + \frac{1}{(n+1)^2} + \ldots + \frac{1}{(2n)^2} \right).$$

[Express as a Riemann sum - compare to examples done in class.]

**Challenge problems:**

**5.** Recall that we have shown that the Schwarzian derivative

$$\frac{u'''}{u'} - \frac{3}{2}\left(\frac{u''}{u'}\right)^2$$

is 0 for every function $u = (ax+b)/(cx+d)$. Now suppose $u$ is any function whose Schwartzian derivative is 0.

(a) Prove that $u''^2/u'^3$ is a constant function. [Differentiate.]

(b) Prove that $u = (ax+b)/(cx+d)$. [Take the square root of the expression of (a). This is still constant. Put $v = u'$ and integrate the new expression $dv$.]

**6.** (a) Prove that

$$\arctan x + \arctan y = \arctan\left(\frac{x+y}{1-xy}\right).$$

["Cheaper solution": substitute $x = \tan\alpha$, $y = \tan\beta$ and apply tan to the formula. Prove the resulting trig. identity using the ones you know. More honest solution: Define $\arctan(x) = \int_0^x \frac{dx}{1+x^2}$ and use the properies of Riemann integral.]

(b) Prove that

$$c \arctan x = \arctan\left(c.\frac{x}{1-x^c}\right)$$

for $x > 0, c > 0$.