MATH 395 PROBLEMS 1 - REVIEW PROBLEMS

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Regular problems:

1. Suppose a matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has an inverse A^{-1} (i.e. a matrix such that $AA^{-1} = A^{-1}A = I$). Find an explicit formula for A^{-1}.

2. Let $S = \int_0^1 e^t/(t+1)dt$. Express the values of the following integrals in terms of S:
(a) $\int_{a-1}^{a} \frac{e^{-t}}{t-a-1}dt$.
(b) $\int_0^1 e^t ln(1+t)dt$

3. Let $f(r,t) = t^n e^{-r^2/(4t)}$. Find a value of the constant n such that f satisfies the following equation:

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r}\right).$$

4. Evaluate the following infinite sums:
(a) $\sum_{n=0}^{\infty} \frac{1}{2^n}$.
(b) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^{2n} \pi^{2n}}{(2n)!}$.
(c) $\sum_{n=0}^{\infty} \frac{1}{3^n(n+1)}$.

Challenge problems:

5. (a) Prove that the series

$$\sum_{n=0}^{\infty} 2^n \sin \frac{1}{3^n x}$$

converges uniformly on $[a, \infty)$ for $a > 0$. [Hint: $\lim_{h \to 0}(\sin h)/h = 1$.]

(b) By considering the sum from N to ∞ for $x = 2/(\pi 3^N)$, show that the series does not converge uniformly on $(0, \infty)$.

6. Find the set of all points $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \in \mathbb{R}^3$ for which the two spheres $(x-a)^2 + (y-b)^2 + (z-c)^2 = 1$ and $x^2 + y^2 + z^2 = 1$ intersect orthogonally. (Their tangent planes should be perpendicular at each point of intersection.)