MATH 395 PROBLEMS 9

IGOR KRIZ

Regular problems:

1. Solve:

$$y' = Ay$$

where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

2. Solve:

$$y' = Ay + b(x)$$

where

$$A = \begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}, \quad b(x) = \begin{pmatrix} x \\ 1 \end{pmatrix}.$$

3. Using Fubini's theorem, calculate

$$\int_{[0,1]\times[0,1]} xy(x+y)dxdy.$$

4. Using Fubini's theorem, evaluate

$$\int_{[0,t]\times[1,t]} y^{-3}e^{tx/y}dxdy,$$

where $t > 1$.

Challenge problem:

5. Let $I = [a_1, b_1] \times \cdots \times [a_n, b_n]$. Prove that if f is bounded on I and continuous on $(a_1, b_1) \times \cdots \times (a_n, b_n)$, then the Riemann integral

$$\int_I f$$

exists. [Generlize the case $n = 1$ which was done in class at the beginning of LDE's.]