MATH 396 PROBLEMS 13

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Regular problems:

1. Using the Lagrangian approach to mechanics, solve the motion of one particle in one-dimensional space with constant homogeneous force field. In other words, find a function $y : [a, b] \to \mathbb{R}$ with given $y(a)$ and $y(b)$ which minimizes the integral of "energy"

$$\frac{1}{2}m(y')^2 + mgy$$

with time ranging from a to b. [Tip: the "simplified" Euler equation in this case leads to a more complicated computation, but a more explicit answer.]

2. Prove that if $f(x+iy) = f_1(x+iy) + if_2(x+iy)$ is a holomorphic function, $f_1, f_2 : \mathbb{C} \to \mathbb{R}$, then both f_1 and f_2 are *harmonic*, which means that

$$\frac{\partial^2 f_1}{\partial x^2} + \frac{\partial^2 f_1}{\partial y^2} = 0,$$

and similarly for f_2. [Use the Cauchy-Riemann conditions.]

3. Let U be a connected open set in $\mathbb{C}$. A function f has the property that

$$\int_L \frac{f(\zeta)}{(\zeta - z)^2} = 0$$

for every circle L with center z where L is contained in U. What can you say about f?

4. Find the geodesics on the cylinder $\{(x, y, z) | x^2 + y^2 = 1\}$. [Parametrize your curve as $(x, y, z) = (\cos\theta, \sin\theta, u)$. Treat θ as an independent variable, u as a function of θ. Write the "simplified" Euler equation for the variation problem minimizing the integral of the derivative of length: $(\frac{dx}{d\theta}, \frac{dy}{d\theta}, \frac{dz}{d\theta})$.] Is there an elementary explanation of your answer?

Challenge problem:

5. Determine the geodesics on the parabolloid $\{(x, y, z)|x^2 + y^2 = z\}$. [Parametrize $(x, y, z) = (r\cos t, r\sin t, r^2)$.]